The interplay of language and mathematics

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This article deals with an intervention unit which focused on the positive effect of integrating poems, stories and fables (the literary genre) for improving mathematical language, fostering the teaching of mathematics language, increasing satisfaction with the process and strengthening the relationship between use of daily language and mathematics language. The study was conducted amongst pre-service mathematics teachers, specialising in literacy activities designed to foster mathematics language. One of the study outcomes was a 'bank' of words with double meanings which can result in misguided perceptions and common errors. Another outcome was a pool of ideas for literacy activities in mathematics which can develop wide, integrative activities. Exposure to and awareness of such a bank of words may help mathematics education practitioners to cope with difficulties in mathematics teaching and learning.

Introduction

Current literature supports the existence of four language components in learning mathematics: reading, writing, speaking and listening (Timor & Patkin, 2010). In the teaching and learning of mathematics one should consider that mathematics has its own unique language, and that learners find some of the mathematical terms ambiguous. There are words which are used in everyday life as well as in mathematics language, but their meaning in each is different or 'double'. In many cases natural language – the spoken everyday language – is blended into mathematics language. Consequently, students 'drag' the meaning of words in everyday language into the mathematics language, which often results in mistakes and misconceptions.

For many years only reading and writing of mathematical symbols were used during mathematics lessons, without any language skills required. This has changed in the last 20 years. The idea of communication in mathematics is expressed clearly in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM), 2000, p. 60): 'They [*learners*] communicate to learn mathematics and they learn to communicate mathematically.' If we re-examine the definition of language (*Webster's Universal College dictionary*, 1997), it seems that mathematics language is a formalised set of conventions that are used for the specific purpose of problem-solving.

This study attempted to examine the interplay between mathematics language and natural language (in this study Hebrew, the formal language of studies at the college of education concerned). The research involved a group of pre-service mathematics teachers who were exposed to the new attitude in mathematical pedagogy, emphasising mathematical discourse and communication as well as views of mathematics as a language.

The pre-service teachers were asked to develop additional materials to help them whilst teaching mathematics in their classes.

Theoretical background

Daily language and mathematics language

This article focuses on the interplay between two languages: everyday, natural language and mathematics language. This interplay is 'performed' by using the natural language in mathematics, integrating literacy activities for the purpose of fostering mathematics language and enhancing the relationship between these two languages through stories and poems.

The poem *Look, we are two numbers* (free translation and emphasis by D.P.) by Israeli poet Yehuda Amichai is an example of a literary composition which includes mathematical terms:

Look, we are **two numbers**,
Standing together and **being added**
or subtracted, because finally **the sign**
changes from time to time.
It was so difficult until we succeeded
to stand together, and we also knew
multiplications of happiness, **fractions** too,
as usually happens to **numbers.**
Even now, under us, the world is a **fraction line** –
don't be alarmed, look how beyond
that **line** has now bloomed for us –
the common denominator.

(Amichai, 1975, p. 36)

This is a love poem describing being a couple. Amichai made surprising use of the mathematics language and created original images for defining the relationship of a couple with its fluctuations 'from time to time'. In describing the relationship he included metaphors, all taken from the world of the basic four operations of mathematics. The mathematical expressions mentioned in the poem embody opposites, reflecting contrasts in human and emotional relationships. He employed mathematical terms as 'scaffolds' for the everyday language. According to him, these terms are not merely mathematical, they also have a meaning in daily life. Applying words by moving and passing back and forth from the everyday to the mathematics language might enhance mathematics language and consequently students' achievements in this field. Mathematics language should be fostered in various ways for that purpose.

Webster's Universal College dictionary (1997) provides a number of definitions of the term 'language', two of which can be related to the definition of mathematics as a language. The first deals with communication using a system of arbitrary vocal sounds, written symbols, signs or gestures in conventional ways with conventional meanings. The second is any set or system of formalised symbols, signs, sounds or gestures used or conceived as a means of communicating. These definitions address the concept of language as a formalised or arbitrary means of communication comprising specific terminology such as signs, graphic representations, numbers and icons.

One should bear in mind that every language is based on unique symbols and syntax rules (*Webster's Universal College dictionary*, 1997) which define the use of its symbols, which can serve to represent meanings in a specific area. If we relate to mathematics as a language built of symbols and syntax rules used for describing objects, actions and relations between sizes, it can be utilised both in writing and orally. This is done both by numerical representations and graphic or iconic representations for abstraction of reality, for use in axioms and in a linguistic-verbal representation. Hence, it becomes necessary to understand the mathematical word lexicon and terms in the teaching and learning process.

Mathematics encompasses unique linguistic forms. There is extensive use of key words which imply the four operations of mathematics (addition, subtraction, multiplication and division), and since the language of mathematics comprises symbols and signs, failing to distinguish between them on visual grounds may impede learning. For example, there can be confusion between 2 and 5, 6 and 9, the signs > and <, or between + and -. Moreover, there is a mix of various meanings of words used in everyday life on the one hand, and in mathematics lessons on the other (as in the poem of Amichai). All of this is difficult and confusing for learners (Cobb, Yackel & McClain, 2000).

Difficulties in mathematics

Throughout the school years many students have difficulties in learning mathematics. In the last two decades the prevailing attitude amongst mathematics educators has been that mathematics is a language and therefore all language components must be taken into consideration in mathematics teaching (NCTM, 2000; Skemp, 1972). Difficulties stem not only from failure to acquire mathematical knowledge but from the use of language in general and the abstract nature of mathematics language in particular (Patkin & Gazit, 2011).

Current literature supports the existence of four language components in mathematics learning: reading, writing, speaking and listening (Kazemi, 1998; NCTM, 2000; Siegel, Borasi, Sanridge & Smith, 1996; Usiskin, 1996). Despite the fact that mathematics is not perceived as a spoken language, the new standards for teaching mathematics indicate the need for verbalisation ('mathematical discourse') (Huinker & Laughlin, 1996).

Skemp (1972) argues that many difficulties and problems in mathematics education are due to the fact that many words have different meanings in different languages, and some words have different connotations in different countries. There are also some words which have double or even triple meaning in the same language. Thus, confusion and obstacles are to be expected. Skemp employed the French term *faux amis* to describe this, which means 'false friends'. He gives the example of the word 'biscuit', which in the United States of America refers to a different item of pastry than in Britain. In the USA one should say 'cookie' in order to obtain the same item, 'biscuit', as in Britain.

Hence, during their lives learners acquire a variety of words, which they apply in order to express thoughts and ideas. Sometimes they use words which represent mathematical terms, whose 'daily' meaning is not in line with their mathematical meaning. All over the world, everyday words in English (such as field, group or set) are included in mathematics language – and this turns them into 'false friends'. In Hebrew, for example, the word 'cube' has a different meaning in mathematics language and in daily life. When you tell a child in kindergarten to bring a cube from the cubes corner, this refers to one of the differently shaped game blocks with which they can build towers, bridges, et cetera. In mathematics language, however, this word refers unequivocally to a cube: a solid figure with six identical square faces, all right-angled.

According to Usiskin (1996) the reasons for experiencing problems in mathematics learning are related to aspects of its language. The fact that students are not exposed to the 'language of mathematics' at home or in their close environment makes teaching harder. It is important to start teaching mathematics at an early stage, so that it can become a mother tongue or second language for the learners. Moreover, mathematics is often taught out of context. This makes learning meaningless and inapplicable to the learners' lives. As a result, the learners study a 'dead', useless language. In addition, teaching mathematics comprises the use of abstract concepts which are not always clear and meaningful to the learner. Usiskin argues that mathematics has much in common with other languages because of the following:

1. Mathematics does not only describe ideas, but also fosters the organisation of these ideas within the learner.
2. The number of symbols and signs in mathematics (e.g. ⊥, = , ≅) is similar to the number of letters in other languages
3. Mathematics has its own syntactic rules, with expressions such as '3+4', and verbs (e.g. subtract).
4. Mathematics has a 'private property' of vocabulary like any other language, as well as its own unique features.
5. Mathematics lends and borrows words, like any other language. It makes use of the Latin alphabet in algebra, the Greek alphabet in geometry (e.g. ellipse, parabola), the word 'radius' from Latin, et cetera. It also lends words; for example, the word 'triangle' in mathematics refers to a two-dimensional shape whereas in everyday English it is also used to describe a romantic loop with three people involved.

The linguistic approach, which illustrates the difficulties in learning mathematics and the perception of mathematics as a language with all its aspects, has a strong relation to literacy activities (Patkin, Millet & Ezer, 2001).

Literacy activities in teaching and their integration in mathematics lessons

The concept that literacy can be enhanced in all learning subjects and that it promotes thinking in the studied discipline was developed in the 1990s (Fulwiler & Young, 1990; Kazemi, 1998; Norwood & Carter, 1994). According to this concept, there is a tendency to integrate literacy activities such as writing, speaking and reading in every subject, in view of expanding relations between disciplines and extracurricular systems. This approach emphasises that comprehension of a complex subject is accomplished by its presentation within multidisciplinary systems.

The professional literature underscores that literacy activities can also be integrated into disciplines not related to the field of humanities. At the beginning of the 1980s Evans (1984) discussed the integration of writing activities in primary school mathematics lessons. She demonstrated that the attainments of students who participated in such a project were higher than those who did not. Moreover, Rose (1989) described a design to integrate literacy in mathematics at high schools in California, illustrating that writing in mathematics lessons allowed students to advance at their own pace whilst using their everyday language and personal experiences.

In recent years mathematics educators have encouraged practicing teachers and their students to develop, in addition to writing, a 'mathematical discourse' during mathematics lessons (Kazemi, 1998). They also advocate use of a reflective written record in order to promote mathematical thinking and learners' ways of coping with mathematical material (Hart, Schultz, Najee-ullah & Nash, 1992; Norwood & Carter, 1992, 1994; Schiebelhut, 1994).

A study amongst different populations of mathematics teachers illustrates that integration of literacy in teaching may contribute to understanding of this subject and reduce fear of it (Ezer, Patkin & Millet, 1999). The question in a study which focused on integration of the fable *The Lion and the Hunter* by Lafontaine was: 'What is the relation between Lafontaine's quarter and the rational number quarter? Or what is the difference between mathematical justice and the jungle justice?' In Lafontaine's fable, four animals went hunting and agreed to divide the prey into four equal parts. According to mathematical justice, each animal was supposed to receive 1/4 of the prey. The lion, however, took everything because he made a 'just' division, according to the jungle laws. Thus, although the animals agreed about equal division, all of the parts went to the lion.

Findings indicated that the use of an intriguing fable that stimulates mathematical thinking led to an educational lesson which promoted a positive approach to mathematics teaching and use of mathematics language (Millet, Patkin & Ezer, 2002).

The present study explored a bank of words with multiple meanings and literacy activities by means of an intervention unit. The research questions addressed in this article are:

1. Which literacy activities can pre-service teachers create or use in order to be more aware of the problems with language and mathematics?
2. How do the reflections of the pre-service teachers provide insights into the importance of integrating literacy activities in the teaching of mathematics?

Research method

This study used qualitative methods; diaries and outcomes activities were qualitatively analysed, allowing thorough and valid research. The use of several data sources enabled triangulation and validation of the data (Payne, 1999).

Methodology

Research population

The study was conducted at a college of education in Israel. It involved 22 pre-service teachers, 10 of them specialising in primary school and the others in junior high school mathematics.

Research procedure

The intervention unit and its stages: The intervention unit was conducted as an annual course dealing with 'mathematics teaching and assessment', with 30 weekly sessions each 2 hours long, 15 sessions in each semester. The course focused on observations of the pre-service teachers's recorded mathematics lessons as well as the investigation and analysis thereof. Preparation of the theoretical basis of the course was grounded on two parts: reading about uses of everyday language in mathematics lessons, and reading about a selected topic in mathematics. During the course the pre-service teachers were asked to write a diary describing their feelings, difficulties they encountered throughout the year, dilemmas, ideas and reflections. The aim was to monitor their feelings stemming from their experience with the course and its products. The diaries were qualitatively analysed.

The first semester: The first session began with the poem of Amichai. The students were divided into small working groups of three or four people. In every subgroup one student had to read the poem aloud (in Hebrew). The second step in this working group was to write the poem in their own words, and then to mark all words in the poem which represented mathematics concepts and to explain the meaning of these words in the poem.

In the second session the students had to check the meaning of these words in a regular dictionary and in the dictionary of mathematics, and compare them to their explanations and definitions. In the following sessions each pre-service teacher had to choose a mathematical topic from the curriculum, and they were asked to read papers discussing the uses of everyday language in general and in mathematics in particular, for example communications in the language of mathematics (Kazemi, 1998; Rose, 1989; Schiebelhut, 1994; Usiskin, 1996). They were also requested to read studies dealing with the teaching of the chosen topic, its historical background, and related misconceptions and common mistakes.

The activity during the first semester consisted of consolidating the pre-service teachers in the two topics and exposing them to use and integration of literacy activities in teaching. They were taught to operate a programme which introduced literacy activities into mathematics lessons, namely leading mathematical conversations (verbalisation of mathematical relations), verbalisation of thinking processes in order to identify misconceptions and barriers, writing and reading students' reflective written records, writing procedure assessment, performing inquiry assignments, developing strategies for understanding mathematical texts and interviewing students. At the end of each session all the pre-service teachers had to indicate all words with a different or double meaning in a list.

Semester break: The pre-service teachers were required to look for a short story, poem, folk tale or fable which included words in Hebrew with a double or triple meaning or one meaning in everyday life and another as a mathematics concept, and to write down the mathematics concepts in the chosen text. Then they had to choose two concepts and look for the theoretical background, including the component of the history of these concepts, the place of these concepts in the curriculum, and student mistakes and misconceptions relating to these concepts. They also had to design an intervention which comprised four to five lessons on the chosen topic using and integrating literacy activities.

Second semester: The pre-service teachers had to practice implementation of the programme in their own classes with children. The weekly sessions consisted of discussions, reflection and analysis of findings, that is executing the programme from theory to practice.

Ethical considerations

Permission to conduct the study was granted by the college of education. The pre-service teachers gave their consent to participate in the study. The aims and objectives of the study were discussed with them. The names of the pre-service teachers were not divulged. The diaries were stored away safely by the author.

Findings

Additional activities and materials

One of the course objectives was to expose pre-service teachers to the linguistic approach to mathematics teaching (Timor & Patkin, 2010). This was achieved by using literacy activities designed to foster mathematics language, enhancing the pre-service teachers' awareness of words with one meaning in everyday language and another in mathematics lessons. One of the main products was a bank of words in Hebrew with one meaning in everyday language and another in mathematics language, together with their definitions.

Throughout the course the pre-service teachers gathered an extensive collection of words which have one meaning in daily life and another in mathematics language. Table 1 shows examples from this 'double-meaning words bank' of the pre-service teachers, representing words with double meaning in each branch of mathematics: arithmetic, algebra, geometry, solid geometry, statistics, differential and integral calculus, et cetera. Their exact definitions were taken from *Chambers dictionary* (Magnusson, 1993) for everyday usage and the *Penguin dictionary of mathematics* (Nelson, 1998) for the mathematical definitions. It is important to mention that amongst these words are some which have a double meaning in mathematics and a double or triple meaning in ordinary language (e.g. 'base').

Using the double-meaning words from the 'bank', discussions were conducted about the ways these words increase the need for accurate definitions of terms as well as the ability to distinguish between the different meanings and definitions which these terms sometimes have. The pre-service teachers and students in their classes reached precise definitions of concepts. For example, one of the pre-service teachers developed an intervention unit with the concept 'height', which has a different meaning in mathematics

TABLE 1: Examples from the 'double-meaning words bank' of the pre-service teachers.†

Word	Everyday meaning	Meaning in mathematics language
Altitude	Height, especially above sea level	Length of the perpendicular from apex to base of a triangle
Axis	A line about which a body rotates	A reference line associated with a geometric figure
Base	Home or headquarters (e.g. of a fleet, a battalion)	The number represented by the numeral '10' in a positional number system, or a line or plane in a geometric figure relative to which the altitude of the figure is measured
Cardinal	Of fundamental importance; a high rank in church	A number that indicates the number of elements in a set
Common denominator	Something that makes comparison, agreement, communication, between things or people possible	A common multiple of the denominator of two or more fractions
Degree	A unit of temperature	The exponent of a variable in a term
Division	A portion or section; one of the parts of a business	The inverse operation to multiplication
Fraction	A breach of friendly relations (Shakespeare)	A quotient of one number by another, or a quotient of two numbers
Function	Duty peculiar to any office or job	A variable so connected with another that for any value of the one there is a corresponding value for the other
Grade	A yearly stage in education (e.g. first grade)	The less common system of angular measure, in which the right angle is divided into 100 degrees, the degree into 100 minutes and the minute into 100 seconds. In this system the degree is also called the grade (or grad), the minutes being the centigrade
Integral	Entire or whole	Describing or denoting an integer
Model	A person posing for an artist or exhibiting clothes in a fashion show	Any system of definitions, assumptions and equations set up to discuss particular natural phenomena
Negative	Denying, expressing denial	A real number which is less than zero
Odd	Strange	An integer that is not divisible by 2
Order	Things arranged in proper condition; a command	The number of vertices of a graph; the number of times a differentiation is performed
Pi/Pie‡	Pie, savoury or sweet dish	Pi - the ratio of the length of the circumference of a circle to the diameter
Power	The authority to do something	The result of multiplication; for instance, 9 is the second power of 3
Prime	First in order of time, rank or importance; of the highest quality	A whole number larger than 1 that is divisible only by 1 and itself
Reflection	Attentive consideration or contemplation	A transformation involving a mirror line or axis
Root	The underground part of a plant	A number that when produced for the variable in a given equatio, satisfies the equation (i.e. makes both sides equal)
Set	Covering the table with food, dishes and cutlery for a meal	A collection of any kind of objects. The objects that make up a set are called its elements or members
Side	Any party, team, interest or opinion opposed to another	One of the lines joining the vertices of a polygon
Solid	Having the particles firmly cohering (as opposed to fluid or gas)	A three-dimensional geometric figure, e.g. a polyhedron or cone
Solution	Mixture made up of one substance that has dissolved into another	Giving an answer to a problem
Square	An open space in town along with its surrounding buildings	A geometric figure having four equal sides and four right-angles
Table	An article of furniture	A compact scheme of numerical information, words, facts, et cetera
Term	Any limited period; a division of the academic year	Part of an equation or mathematical expression
Triangle	A musical instrument of percussion	A plane figure with three angles and three sides

†, It is important to mention that the word bank includes examples of words which have double meanings in Hebrew as well as in English.
‡, In this case, the words sound the same but are spelled differently.

language (the height of a triangle, etc.) and in daily life. For example, in Hebrew when somebody asks what the height of the building is, he means its length. Conversation about this word led to inquiry and the conclusion that height exists in triangles (there are three heights), but not in every polygon. All of this can be used as a tool for reducing mistakes in mathematics

Another example which led to a very interesting activity was the word 'division' (e.g. when eight candy bars have to be divided between four children). In daily life the division must not necessarily be into four equal parts; there might even be an extreme situation where one child does not get any candy whereas another receives all eight bars. Conversely, in mathematics, if we divide eight candy bars between four children the tacit assumption is that the division is into four equal parts. Activity of this type is similar to that presented in the study of Millet, Patkin and Ezer (2002), which focused on a literacy activity in mathematics on the basis of Lafontaine's fable, described earlier.

Another outcome was a bank of fables, poems and other types of literature which can be used in intervention units or sessions for the interplay of everyday language and mathematics language, such as fables by Aesop and Lafontaine, stories from the Bible, et cetera.

Findings obtained from diary analysis

The diaries were completed in free writing during the intervention unit (two semesters, including semester break). The pre-service teachers willingly wrote in the diaries and were pleased to share their feelings. Twenty of the 22 participating pre-service teachers actively wrote in the diaries, indicating mathematical terms which they found difficult, experiences during the intervention unit, and words with various meanings which were new to them. Consequently they concluded that perhaps the different meanings of the words of which they had not been aware before constituted a reason for misunderstanding and misconceptions of these terms.

Analysis of the diaries revealed four key indices, namely pleasure, comprehension of the importance of knowledge about double meanings, a positive attitude towards the continuation of teaching in such a way, and a sense of self-confidence. The following excerpts from the diaries provide examples of each index:

Pleasure

> I am very satisfied. They (the students in my own class) have never believed that they will have to tell and explain words. They were accustomed to solving problems and that was all. Talking about reflective writing in mathematics was non-existent because it might have wasted precious time in the lesson ... But now I have realised that with some imagination one can plan many lessons filled with surprises which are growing from the students' natural language, helping them to express their ideas. One of my students told me that: 'I knew, but now I also understand'.
>
> [Pre-service teacher R]

Comprehension

> By means of the activity which transcended mathematics, I reached additional areas with the students. The studied mathematical subject was height of triangles. One of the students asked what the height of the Eiffel Tower is and a second student answered that in order to measure this we must know which measurement units to use and must make an estimate. On the other hand, when we dealt with the concept of height of a triangle from the mathematical point of view, we examined the options of the place of height in all triangle types. The activity enhanced the difference in the interpretation of the word 'height'.
>
> [Pre-service teacher R]

Positive attitude towards the continuation of teaching in such a way

> I felt that my teaching level had changed extremely, both from the point of view of using the means concerning the students and from my personal feeling, because I helped the students to acquire tools for coping with general knowledge and not merely with specific knowledge.
>
> [Pre-service teacher L]

> I have undergone a change and I think that I am beginning to understand what is going on. The discussions and talks conducted during the entire academic year constituted part of my professionalisation process ... Until several days ago it was difficult for me to internalise and understand what was transpiring in my class. However, now when I am writing I actually think that one of the problems in my class stemmed from the fact that the students were not exposed to words with different meaning in different languages like the term 'to reduce'. In Hebrew as in English, to reduce means making it smaller, to reduce weight etc. … in mathematics, to reduce means to write equal fractions in other way.
>
> [Pre-service teacher M]

Sense of self-confidence

> … the activity in which we engaged during the course gave me a lot of power. Power to understand that we can make a change ... Acquaintance with mathematics as a language is a gift for life … We have learnt to try everything, as new as it might be, and particularly to continue and persevere with the change, not to be afraid of it. Thank you for a real change in the way of teaching objects in mathematics. It has displayed and will continue displaying its fruits in the field.
>
> [Pre-service teacher D]

> I indicated that all the class students as well myself sensed a change following the talks about words with double meaning and the activities.
>
> [Pre-service teacher E]

> … asking the students to bring examples of words with different meanings and explain them as part of the integration of literacy activities in lessons throughout the course, made me sense that the concept which was taught became clear and comprehensible.
>
> [Pre-service teacher Y]

To summarise, the findings indicated progress in terms of comprehension and ability to define words and terms correctly. Moreover, the pre-service teachers indicated a sense of pleasure resulting from progress achieved in the course as well as enhanced motivation to teach in this way.

Discussion and conclusions

This study describes the interplay of language and mathematics amongst pre-service teachers in elementary and junior high school.

An examination of the literature supports the existence of four language components in mathematics learning (Usiskin, 1996; Uso-Juan & Martinez-Flor, 2006). This study involved a group of pre-service mathematics teachers who were exposed to the new attitude in mathematical pedagogy, which emphasises mathematical discourse and communication and views mathematics as a language requiring use of oral and written components (Timor & Patkin, 2010).

The intervention unit exposed the pre-service teachers to numerous words with double or triple meanings which were 'false friends'. When teaching a mathematical subject comprising words which have different meanings in everyday use and in mathematics, they discussed these words with their students. This process made them more attentive and sensitive to the possibility that double or triple meanings caused problems in the comprehension of their students.

Throughout the entire year whilst teaching in their classes they searched for words with different or confusing meanings, adding words from their own personal knowledge. The search for this word lexicon was performed by means of all the literacy activities described above. The students also directly addressed the students, asking them to search by themselves for short stories, poems, et cetera, to present them orally or in writing and to emphasise words with one or more meaning in everyday life and another in mathematics language. Integrating literacy activities into mathematics by raising awareness of the various meanings that words can have can, as reported in the diaries, eliminate mistakes (NCTM, 2000; Patkin & Gazit, 2011).

Speaking plays an important part in learning. It requires students to organise their thoughts and to focus them. Mathematical speech, which is a combination of everyday language and mathematics language, is manifested by reflective mathematical talks (Usiskin, 1996). As one of the pre-service teachers reported in his diary:

> The talks about the different meanings helped me to be more accurate when making definitions and it influenced my way of teaching. It helped my students construct their mathematical comprehension and apply in practice their theoretical studies …
>
> [Pre-service teacher L]

Discussing questions like 'Is it correct to refer to the wooden blocks from which kindergarten children build towers as "cubes"?' can demonstrate to the learners the risks embodied in failing to be meticulous in the application of words. There is no problem calling this section of the kindergarten the 'cubes corner' or the 'solid figures corner'. One might say that we should not teach young children (up to age three years) the mathematical vocabulary because it is too difficult for them. However, just as young children learn to differentiate between different colours and the names of many animals, it is important to emphasise accuracy in mathematics terminology, otherwise they develop misconceptions and mistakes with these concepts.

We should take advantage of the opportunity to actively construct mathematical knowledge through mathematical talk, by means of which we identify difficulties, compare solutions, ask leading questions and present assumptions for discussion and thought (Siegler, 2006). Encouraging students to make use of verbalisation, writing reflective records, writing procedures and building stories which include mathematical problems creates conditions for the learning of mathematics whilst participating in social class processes.

During the course the pre-service teachers attested that the literacy activities in mathematics lessons not only enriched their personal knowledge but also extended their mathematical thinking. Using everyday language as scaffolds assisted them to teach the 'second' language – mathematics language. The course outcomes reinforced and enhanced the need for teaching of this kind. For example, the poem of Amichai (1975) 'crossed' the border and made use of mathematical terms, assuming that readers are familiar with these. The poem describes addition, subtraction and multiplication; it discusses fractions, two numbers, fraction line and common denominator. In the poet's mind these terms are not merely mathematical terms but rather words with a double meaning.

To summarise, it is possible and recommended that learners be encouraged to move back and forth from one language – natural, everyday language – to a second language, mathematics language. Thus we can promote mathematical thinking and comprehension and, as a result, improve students' attainments in this discipline. It is recommended that research of wider scope (including quantitative data) be conducted to validate the findings of this study.

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Competing interests

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