Exploring teachers' conceptions of representations in mathematics through the lens of positive deliberative interaction

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This article reports on an exploration of teachers' views on the meaning of mathematical representations in a democratic South Africa. We explored teachers' conceptions of 'mathematical representations' as a means to promote dialogue and negotiation. These conceptions helped us to gauge how these teachers viewed representations in mathematics. Semi-structured questionnaires were administered to 76 high school mathematics teachers who were registered for an upgrading mathematics education qualification at a South African university. Common themes in teacher conceptions of representations were investigated as part of an inductive analysis of the written responses, which were considered in terms of practices that support dialogue and negotiation. Findings suggest that these conceptions are in line with progressive notions of classroom interactions such as the inquiry cooperation model. Furthermore, the findings suggest that teachers can support the development of classroom environments that promote democratic values.

Introduction

What are the specific elements of a mathematics classroom that allow it to be characterised as democratic or as a classroom that seeks to prepare all children for life in a democratic society? Are there connections between democracy and mathematics classrooms? Skovsmose (1998) asserted that mathematics education could be related to a discussion of democracy in terms of citizenship, mathematical archaeology, mathemacy and deliberative interaction. He illustrated how these four aspects, which concern classroom practice in mathematics education, also concern democracy. In this article we delve into one of the above aspects, namely deliberative interaction, which Skovsmose views as possible when 'an interaction in the classroom which supports dialogue and negotiation' is developed (p. 200). Figure 1 illustrates the components inherent in Skovsmose's notion of deliberative interaction.

Deliberative interaction excludes a view of mathematics as an unchanging body of knowledge, which a teacher transmits to learners. Such a view presupposes that mathematical tasks have only one correct answer and often only one correct, or one preferred, method to arrive at that answer. This view sets the classroom as an autocracy in which the teacher serves as the sole authority. Alrø and Skovsmose (1996) used the phrase *classroom absolutism* to refer to the type of communication between the teachers and learners that is structured by assumptions that (1) school mathematics can be organised around mathematics activities with unique answers, and (2) the teacher's task is to ensure that mathematical errors are removed from the classroom. In trying to identify the role of representations when teaching and learning mathematics within such a paradigm, we extend the notion of classroom absolutism to include two further assumptions, (1) that mathematical learning can be organised around classroom activity with one possible representation for a mathematical notion or task, and (2) it is the duty of the teacher to ensure that other representations are eradicated from mathematical learning.

In the democratic micro-society of a mathematics classroom it is imperative for the teacher to move away from such classroom absolutism because learners should be afforded different ways to express themselves. We are not implying that teachers accept any and all responses to mathematical tasks as final answers. Our message is this: the teacher should (1) be aware of the different mathematical representations that can be used to achieve mathematically acceptable arguments, and (2) be willing to work with learners' developing mathematical ideas and personal mathematical representations to facilitate a clearer understanding of mathematics and the way it is conventionally represented.

Educational environments that discourage classroom absolutism often have a prevailing view of mathematics as a process rather than a product. Mathematics is much more than the production

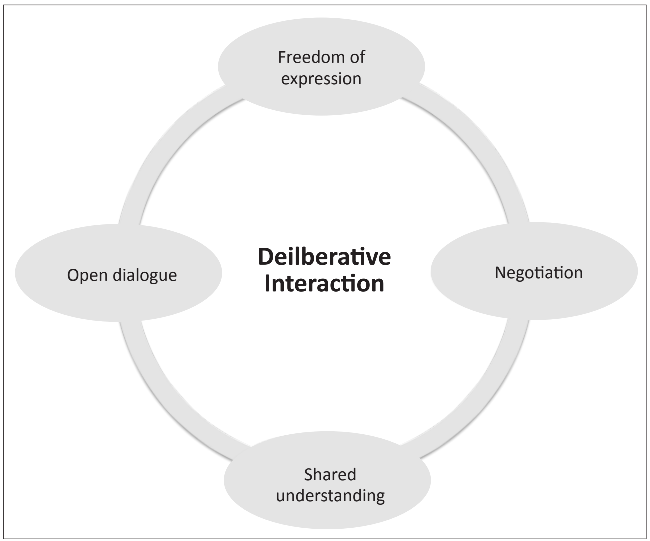

FIGURE 1: Components of deilberative interaction.

of answers; it is the process of determining how to quantify, model, et cetera a situation. Rather than producing an equation or a table or a graph, educational environments that discourage classroom absolutism should emphasise that different representations of the same mathematical concept are possible and that doing mathematics is often the process of determining what is asked or needed and the affordances and limitations of any mathematical representation that could be used in the answer. Such classrooms exemplify the inquiry cooperation model (Skovsmose, 1998, p. 200), which refers to a 'pattern of communication where the student and teacher meet in a shared process of coming to understand each other' whilst learning about mathematics. Mathematical representations, as vehicles of communication, play a central role in such classrooms. In the mathematics education community, the concept of mathematical representation has been based on different theoretical perspectives (English, 1997; Goldin, 1998; Presmeg, 1997). We adopt the widely used definition that a representation is a configuration that can represent something else (Goldin, 2002).

The representations used to communicate ideas, including those involving mathematical concepts, are socially embedded and culturally created (Greeno, 1997). Therefore, the manner in and extent to which representations mediate mathematical understanding depend as much on the individuals engaged in the task as they do on the task itself. The use of multiple mathematical representations and the fostering of an environment that facilitates and values various representations provide a space where learners can engage with substantial mathematics and develop the tools to become citizens who are productive and active, two qualities of democratic mathematics education (Ellis & Malloy, 2007).

Rationale and research questions

The most recent reform in curriculum and assessment policy in South Africa aims at producing learners who are able to *communicate effectively using visual, symbolic and/or language skills in various modes* (Department of Basic Education, 2011, p. 2). This is expected to occur throughout their learning where opportunity for representation arises. Teachers need to engage in meaningful discourse with their learners so as to better recognise and appreciate the learners' use and understanding of specific representations. Such shared exchanges result in a process in which two groups come to understand the other's viewpoints as well as the discursive resources and mathematical representations employed to communicate those viewpoints. In this way, deliberative interaction is exemplified. However, to build on and from the representations of learners, teachers must have both a deep understanding of the different representations (including the affordances and drawbacks of each) and the flexibility to use the representation that is most appropriate for the mathematical situation and the learners. The issue under investigation in this study is whether teachers have deep, flexible understandings of mathematical representation that enable them to create democratic environments in their mathematics classrooms. This concern will inform mathematics teacher educators and the relevant educational department authorities whether these teachers are prepared for changing educational policies (Department of Basic Education, 2011). Accordingly, this study was designed to explore teachers' understanding of mathematical representations. For this study, we formulated the following research questions:

1. What do teachers in a democratic South Africa believe is meant by the expressions 'mathematical representations' and 'representation in mathematics'?
2. How do teachers view representations in mathematics?

We ask these questions because we believe that the use of a variety of mathematical representations for differing purposes is a powerful tool for teachers to foster deliberative interaction in the micro-society of the classroom. The model presented in Figure 2 is an attempt to show the role of mathematical representations in creating democratic classroom environments.

Teachers need to be able to make flexible use of representations before they are able to create an environment that allows learners the freedom to use developing representations. Thus, teachers' *representational fluency* impacts on their ability to foster deliberative interaction. In this study, 'representational fluency' means that an individual has an abundance of mathematical representations at their disposal for use when reasoning and communicating in the mathematics classroom. In classrooms where deliberative interaction is embraced, learners' communication and representational fluency are encouraged and developed through shared negotiation between and amongst learners and the teacher. Development of representational fluency in learners will better prepare them to interpret mathematical tasks, share their mathematical ideas, and interpret the mathematical communication of others. Hence, representational fluency can contribute positively to developing active citizens, by giving them a sense of freedom of expression, which is a concern of democracy, within the mathematics classroom.

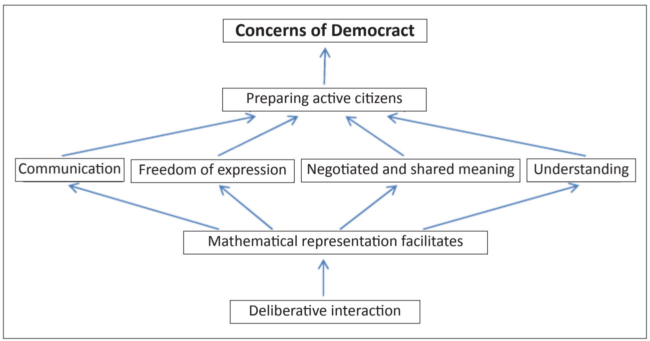

FIGURE 2: Various mathematical representations supporting deliberative interaction in the context of democracy.

With these qualities we hope that the pupils become active citizens, taking ownership of their learning and thus becoming responsible citizens. Responsibility is a prerequisite for upholding democracy. These connections are indicated in the model in Figure 2, which shows the links between the use of mathematical representations and the development of a democratic society.

Classroom communication

Mathematical representations can facilitate dialogue between teachers and their learners, if teachers choose not to conform to classroom absolutism. Of course, classroom communication may be constrained by the pre-described assigned roles of the teacher and the learner (Skovsmose, 1998). However, if the teacher accepts a shared, negotiated dispensation with the learners then the mathematical representations used become ideal entities to promote interactive dialogue. Vithal (1999) argued that the learners in her study who used drawings and graphs to represent their expenses and interviews as part of their project, demonstrated that the classroom 'could serve as the arena for acting out a democratic life' (p. 29).

It is reasonable to suppose that the development of learners' understanding of mathematical ideas and their capacity to use representations to communicate and reason about ideas are influenced by the nature of their teachers' conceptions of mathematical representations. Teachers need to believe that representations can be used as tools to understand mathematical concepts and solve problems but also as modes of communicating about these problems and concepts (Roth & McGinn, 1998). In the sciences, Ochs, Jacoby and Gonsales (1994) studied the work of a group of physicists to display how professionals use representations to create a shared world of understanding. In mathematics education, Moore-Russo and Viglietti (2012) investigated teachers in collaborative problem-solving situations and found that even when presented with the same task, individuals within the groups use various resources to communicate and reason mathematically, often adopting and adapting the representations used by their group members. For teachers to value such situations, they need to foster a democratic classroom environment that departs from classroom absolutism.

Stenhagen (2011) and Allen (2011) have suggested that teachers, teacher educators and curriculum designers place emphasis on teacher beliefs and philosophy in classroom instruction. This article explores teacher beliefs about mathematical representations, with the aim to discover the beliefs teachers have about the practice of teaching in general and the use of representations in particular. The findings should provide insight into the deliberative interactions in mathematics classrooms. Teachers using representations in a way that creates deliberative interactions build possibilities for the classroom to serve as an opportunity for learners to become members of a democratic micro-society and, in doing so, preparing to be active citizens in a democratic society.

Methodology

This study was qualitative in nature. It has been argued that interpretive researchers use mainly qualitative research methods in order to gain a more in-depth understanding of the participants' perceptions of the phenomenon (Cohen, Manion & Morrison, 2007; Henning, 2004). This ties in with our method of inquiry since we intended to find out what teachers in a democratic South Africa believe is meant by the expressions 'mathematical representations' and 'representation in mathematics'. The research instrument used was an open-ended questionnaire. By allowing for free responses, the instrument allowed the research team to elicit the opinions of the teachers without influencing them to provide the answers they felt might please us. A non-probability sampling strategy was used. This is in line with the study because qualitative researchers do not count generalisation as their primary aim but instead seek to represent a particular group (Cohen et al., 2007; Maree & Pieterse, 2007).

The study participants were 76 teachers from historically disadvantaged schools, pursuing an Advanced Certificate in Education, specialising in high school mathematics teaching in Grades 10–12, at a South African university. All had successfully completed the first semester course on Differential Calculus.

The questionnaire

The semi-structured questionnaire, with twelve items, was administered to the 76 participants in the second semester of their study. For this article we consider only the teachers' responses to the first two items of the questionnaire, namely:

- Item 1: What does the phrase 'mathematical representation' mean to you?
- Item 2: What comes to mind when someone talks about 'representation in mathematics'?

The teachers' responses to the two items were analysed for emerging themes through a general inductive analysis. Using theoretical memoing (Glaser, 1998), the research team members individually classified the teachers' responses, then collaboratively developed categories based on their memos. These initial categories established the themes described in Table 1.

TABLE 1: Themes for mathematical representations.

Theme	Description
Examples	Examples of mathematical representations are given (e.g. sketches, diagrams, tables, graphs, equations, verbal scenarios); equations are sometimes described as involving numerals, written words and mathematical symbols.
Representation	Mathematical representations represent, portray, demonstrate or stand for something; they capture mathematical problems, solutions, information, thoughts or ideas in an efficient, effective manner.
Variety	There are various mathematical representations; there are different ways to represent mathematical concepts or ideas.
Communication	Mathematical representations provide a (logical or convenient) way to convey, deliver, present, offer, explain or express mathematical information.
Aid for understanding	Mathematical representations help individuals to interpret, understand or approach mathematics; implicit in education is the idea of developing understanding, so representations are a means to facilitate the teaching and learning of mathematics; the use of additional representations is not needed if a concept is immediately understood.
Real life	Mathematical representations are used to represent real-life situations or scenarios.
Problem solving	Mathematical representations are a means to solve, clarify, simplify, give insight into or analyse mathematics problems.
Tools	Tools or equipment (e.g. chalkboards, calculators) are used to create mathematical representations.
Flexibility	Individuals should be able to work with whichever representation is given; individuals should be able to move between different representations.
Visualisation	Mathematical representations help a person to visualise, picture or illustrate the situation or problem.
Differentiation and selection	Different mathematical representations cater to different types of learners or different learning styles; individuals can or should choose the representation they prefer or best understand.
Interrelation	Mathematical representations are related, interdependent, interchangeable or interrelated; it is possible to move between representations.

The research team used the 12 categories to individually revisit the data to ensure that their constant comparison method constituted a saturation of categories. Using a teacher's response to an item as the unit of analysis, two of the team members independently coded all teachers' responses. Working independently, the two coded each response as providing evidence, or not, for each of the 12 themes shown in Table 2. The 76 teachers' responses to the two items provided 152 units of analysis. The overall inter-coder agreement for the teachers' responses was 0.95; the related Cohen's kappa value was 0.80, above the 0.60 that is accepted to represent good agreement (Altman, 1991; Landis & Koch, 1977). After inter-coder agreement was determined, all disparities in assigned codes initially given to the responses were treated in the following manner: each disparity was identified and then two members of the research team discussed coding until a consensus was reached for each response. The consensus codings were used for all subsequent data analysis.

Once the data set was completely coded, the research team discussed what they saw emerging from the data and collapsed the initial categories into broader themes. The team members then individually revisited the data once more to verify that the themes made sense of the data (Thomas, 2006). Finally, the whole team finalised the descriptions of the 12 themes that were used for data coding.

Issues of ethics and trustworthiness

Ethical clearance was obtained from the university research office for the collection of the data. To comply with the terms of the university research policy, consent to participate in the study was provided by all the participants.

In qualitative research, reliability and validity are conceptualised as trustworthiness criteria (Golafshani, 2003). To eliminate bias and increase researcher truthfulness, triangulation in this study was achieved via independent coding and with agreement being reached by consensus. In addition, the researchers sought convergence of different responses to form common themes from the categories.

Findings

After themes were identified and the data set was coded, the research team generated descriptive statistics to complete the analysis of the data. The first consideration was which categories were most frequently evidenced in the teachers' responses. Information regarding the 12 identified themes that were evidenced in the teachers' responses is summarised in Table 2 in order from most to least common themes.

Note that the columns in Table 2 provide information for each item as well as cumulative information on both items. In order to read Table 2, consider the first row: 32 teachers' responses to Item 1, 52 teachers' responses to Item 2, and 59 teachers' responses to only one of Item 1 or Item 2 were coded as evidencing the *Examples* theme. The data for the *Examples* theme is illustrated in Figure 3.

During the coding process, it was apparent that many teachers' responses provided evidence that the teachers' beliefs regarding representations addressed many of the 12 identified themes. For this reason, details regarding the number of themes noted in each teacher's responses to the two items are provided in Table 3.

TABLE 2: The presence of themes in teachers' responses (N = 76).

Theme	Responses to		
	Item 1	Item 2	Item 1 or Item 2
Examples	32	52	59
Representation	32	12	41
Variety	24	11	30
Communication	15	8	21
Aid for understanding	13	13	20
Real life	9	11	18
Problem solving	10	5	13
Tools	3	7	9
Flexibility	5	1	6
Visualisation	4	0	4
Differentiation and selection	1	1	2
Interrelation	1	0	1

N, number.

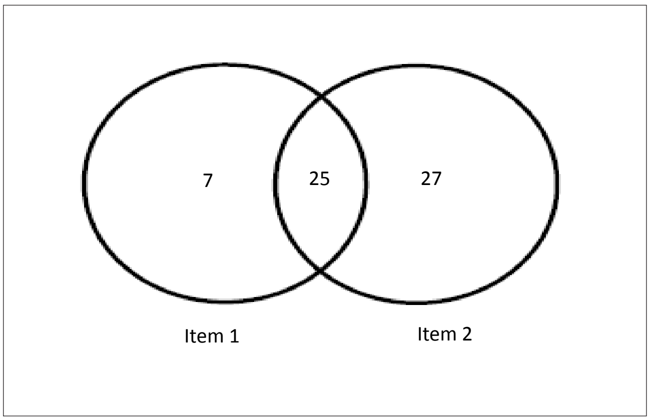

FIGURE 3: Venn diagram showing the coding evidencing the *Examples* theme.

TABLE 3: Number of themes present in teachers' responses (N = 76).

No. of themes in responses	Responses to		
	Item 1	Item 2	Item 1 or Item 2
0	2	3	0
1	20	42	9
2	35	20	16
3	17	5	28
4	2	6	17
5	0	0	5
6	0	0	1
7 or more	0	0	0

N, number.

In order to read Table 3, consider the first row: 2 teachers' responses to Item 1, 3 teachers' responses to Item 2, and 0 teachers' responses to only one of Item 1 or Item 2 were coded as evidencing 0 themes. The final column shows how teachers responded to both items.

Analysis and discussion of data

In this discussion, note that we use the exact responses of the teachers, without editing for language or clarity. The notation T1 is used to denote the first teacher in the list, and T76 denotes the last teacher. We will primarily emphasise the themes that are most pertinent to promoting positive interactions and a democratic environment in the classroom.

The theme *Examples* was most commonly noted: almost 80% of the teachers gave examples of representations in their responses. In considering the *Examples* theme, it is noteworthy that 84 responses (32 to Item 1 and 52 to Item 2) provided examples of representations although only 59 teachers used examples on one item only. This means that a significant proportion (84 – 59 = 25) of the teachers used examples in their responses to both items. Some teachers mentioned only examples of a single representation, for example, T6 wrote 'writing mathematics in graphical form' for Item 1 and 'graphs' for Item 2. On the other hand, many other teachers listed a variety of examples of representations, such as T31:

> here mathematical knowledge is represented using verbal, pictures, symbols and manipulatives.

The *Representation* theme was second most common, addressed by 54% of the teachers. In this theme, teachers described what representations are and how they are used, especially in response to Item 1. A typical example is T14:

> All representations are important based on the concept in which you are dealing with.

The teachers in this sample displayed knowledge of numerous types of representations (as seen in the responses coded under the *Examples* theme) as well as a belief that mathematical ideas can be represented in different ways. This *Variety* theme was the third most common, with 40% of the teachers showing evidence of it in their responses. The *Variety* theme was applied to teachers' responses that explained that there are many different ways to represent mathematical concepts, ideas or relationships. Such responses show that the teachers strongly believe that there are different ways to represent mathematical concepts. This is evidence that these teachers do not subscribe to an absolutist view of classroom communication. One example of the *Variety* theme was displayed by teacher T17:

> Using a variety of ways to capture concepts and relationships. Being able to develop, share and preserve thoughts in mathematics.

What was encouraging, in terms of promoting democracy, was that this teacher perceived mathematical representation as 'using a variety of ways'. This abundance of alternatives is crucial to create a democratic mini-society (classroom) since these 'variety of ways' establish a 'sharing' of mathematical thoughts, which is a positive contribution to a democratic classroom. In this response, teacher T17 does not specify if the teacher or learner initiates this variety of ways of sharing. This could imply that either the teacher or the learner could employ a variety of ways; sharing is thus regarded as a two-way phenomenon, with importance placed on both key players in the classroom. In this case, the teacher would have no dominance, but be regarded as an equal to the learner in the classroom.

The *Communication* theme was noted by 38% of the teachers. Whilst mathematical concepts are important, representations are the vehicles through which these concepts are shared with others. As evidenced by responses that fall under the *Communication* theme, 21 teachers saw representations as things that convey or express mathematical information. An example of a response in this theme is T31's response to Item 1:

> Learners can use the representations themselves to communicate their understanding of the (mathematical) concepts to the rest of the class or in smaller groups.

Here the perception of T31 is that mathematical representation is learner driven. This is in keeping with the principles of the South African school curriculum (Department of Education, 2003). T26 put it another way:

> The way you conveying the knowledge of maths to one another.

We interpreted this as being related to communication since the knowledge is being conveyed to others. This comment also reveals that the teacher does not see the communication as being one-sided; rather it is communication with 'one another'. This view of communication of mathematics ideas as being both from and to the teacher is aligned to the inquiry cooperation model (Skovsmose, 1998) because representations are being used as a form of communication where the learner and teacher meet in a shared process of coming to understand each other, whilst learning about mathematics.

Twenty-six per cent of the teachers had responses that were coded as evidence of the *Aid for Understanding* theme. This reveals that the teachers see representations as facilitating the understanding of mathematical concepts or relationships. T3 expressed the view that mathematical representation means:

> [*a*] way of delivering and presenting the concepts such that the concept is very understandable to learn, encouraging learners to participate willingly and stay in every learner's mind to his life-long period.

It is notable that T3 included the need for mathematical concepts to be made understandable to learners. This provides evidence that this individual is a caring and accountable teacher who wants to make learning accessible for all. This trait is valuable in acknowledging the purpose and function of effective schooling as desired by any democratic society. T3 specifically uses the words 'to participate willingly' and mentions 'every' learner. Two aspects emanate implicitly from T3's response, (1) participation by all learners and (2) freedom of expression. The first aspect evokes the concept of participatory democracy, which requires that all individuals be afforded the opportunity to take part in the decisions that affect their lives (Devenish, 2005). The second aspect alludes to free will, as evidence by T3's use of the word 'willingly'. Freedom of expression is entrenched in the Bill of Rights within the South African Constitution and is fundamental to liberal democracy (Devenish, 2005). Freedom of expression is indispensable in establishing mathematical truth in proof or problem solving, and it is a means of fulfilment of human personality since mathematics is a human activity (Department of Basic Education, 2011, p. 8).

With the recent emphasis worldwide on the need for links between mathematics education and real-life situations, it is no surprise that 24% of teachers identified the role of representations in portraying *Real Life* situations. The linking the learning of mathematics to real life is of paramount importance. This is highlighted in the curriculum and assessment policy statements (Department of Basic Education, 2011), which state in the first specific aim that real-life situations should be incorporated into all sections whenever appropriate. Such linking will prevent the classroom from being a micro-society in which only mathematical abstractions prevail. T51 places emphasis on real life in his response to Item 1:

> Depending on real life situation. One problem might require a graph to solve (a mathematics task), another may require a table, while some may require a flow chart.

T51 indicated that the type of mathematical representation employed is dependent on the real-life situation to which it applies. This shows that this teacher places greater emphasis on the need for contextualisation than on the particular mathematical representation. This could mean that the teacher places the context first in making the choice of which mathematical representation to use to foster the learning of a particular mathematical concept.

In some of the most common responses, teachers mentioned that representations are used for *Problem Solving* (17%) and discussed the *Tools* (12%) that they use to create mathematical representations.

Teacher T61 was one who associated mathematical representations with problem solving:

> Simplify problems by interpreting, analysis using sketches or mind maps.

She also perceived mathematical representation as a means to simplify the problem situation. Her aim to make mathematics problem solving more understandable and, hence, more accessible to her learners indicates her respect for them.

Across the two items, nine teachers associated representations with the *Tools* (equipment or resources) used to create them. For example, T21 wrote:

> … being able to use different approach in sketching, use of computer, …

whilst T46 wrote:

> Mathematical representations refer to visual images which are ordinarily associated with pictures in books and drawings on a overhead projector.

These responses suggest that these teachers see classroom resources and tools as an advantage in trying to present various forms of mathematical representations of mathematical concepts and ideas. They seem to want everybody to have access to tools and resources, a privilege that most, if not all, of the teachers from historically disadvantaged backgrounds in this study were denied.

The remaining themes were *Flexibility*, *Visualisation*, *Differentiation & Selection*, and *Interrelation*.

From the results, it is clear that many of these high school teachers have a rich idea of the roles played by representation in mathematics. Table 3 presents the 12 themes that were discerned in the teachers' responses. From these data we know that (28 + 17 + 5 + 1 =) 51 of the 76 teachers thought of representations in multiple ways since their responses evidenced three or more themes. These results demonstrate that the majority of the teachers had a rich, broad understanding of representations, as opposed to a narrow or limited understanding, and their roles as would be associated with an absolutist view of mathematics.

Conclusion

Despite the teachers being previously disadvantaged, with access to few resources and a varying quality of initial teacher preparation, their views on mathematical representation provide evidence of their willingness to embrace a democratic approach to teaching mathematics. The responses have revealed that many of the teachers see representations as being interrelated and the need to move between representations showed a fluid, dynamic and flexible understanding of mathematics, once more aligned to a democratic classroom.

This abundance of alternatives offered by mathematical representations is crucial to creating a democratic classroom environment since this 'variety of ways' establishes a 'sharing' of mathematical thoughts thus allowing for contributions by both learners and the teacher. The choice of mathematical representation available for classroom activity encourages free will in expression of the relevant mathematical idea. This aspect alludes to an individual's freedom of expression regarding mathematical concepts using mathematical representations.

The use of mathematical representations caters for greater learner involvement and participation during classroom activities, which enhances participatory democracy. The responses of these teachers displayed that mathematical representations are potentially a means of encouraging a form of classroom interaction that promotes dialogue and negotiation in a democratic South Africa.

We are encouraged by the teachers' flexible and open-minded approach to the use of representations in the mathematics classroom. We believe that with the display of mathematics teachers' knowledge of various kinds of representations, and the various ways in which representations can be used in their classrooms, will enhance their teaching practices. Their responses suggest that they see the learning of mathematics as a shared process and not a one-way transmission of a product from the teacher to the learner. The findings from this study also suggest that the teachers want to engage in the inquiry cooperation model (Skovsmose, 1998), rather than following the absolutist tradition, and are keen to use a variety of representations to facilitate understanding of mathematics processes. The findings also showed that the teachers believed that learners and teachers could use representations as a tool for communication and were positive about freedom of expression in their classroom. All of the abovementioned findings augur well for the creation of deliberative interactions by these teachers in their classrooms, which we believe will support the creation of a democratic environment by enhancing the development of active citizens.

More specifically, the data suggest that the teachers believe that mathematical representations can (1) be used to reason and preserve thought in mathematics classrooms, and (2) be used as a tool for sharing thoughts and communicating ideas related to mathematical tasks. Moreover, the study suggests that teachers believe that representational fluency (1) creates opportunities for willing participation by both learners and teachers during mathematics classroom interactions, and (2) aids individuals as they express mathematical ideas freely. These views and beliefs on mathematical representations all facilitate communication, freedom of expression, negotiation and shared meaning, and understanding, which are vital attributes of deliberative interaction, as displayed in Figure 2. These observed attributes are envisaged to prepare active citizens in the mathematics classrooms, thus addressing some of the concerns of democracy.

We are mindful however that the study is based on the teachers' reports of their views of mathematical representations and not on their actual classroom practice, which may not be aligned with these positive reports. Further study should continue in this line of research to determine whether teachers' apparently democratic leanings towards mathematical representations and their uses translate into democratic classroom practices and the facilitation of democratic learning environments.

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Competing interests

We declare that we have no financial or personal relationship(s) that may have inappropriately influenced us in writing this article.

Authors' contribution

The idea to work in this field of representations in mathematics education was encouraged by D.M-R. (State University of New York) and D.B. (University of KwaZulu-Natal) promoted the conceptual framework of democracy and mathematics education for the research design. The creation and implementation of the research instruments were done collaboratively by D.B., S.B. (University of KwaZulu-Natal) and D.M-R. Data collection was carried out by D.B. and S.B. The analysis of data was led by D.M-R. and worked on collaboratively with D.B. and S.B. D.B. wrote the manuscript and it was refined by D.M-R. and S.B.

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