Teachers' reasoning in a repeated sampling context

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The concepts of variability and uncertainty are regarded as cornerstones in statistics. Proportional reasoning plays an important connecting role in reasoning about variability and therefore teachers need to develop students' statistical reasoning skills about variability, including intuitions for the outcomes of repeated sampling situations. Many teachers however lack the necessary knowledge and skills themselves and need to be exposed to hands-on activities to develop their reasoning skills about variability in a sampling environment. The research reported in this article aimed to determine and develop teachers' understanding of variability in a repeated sampling context. The research forms part of a larger project that profiled Grade 8–12 teachers' statistical content and pedagogical content knowledge. As part of this larger research project 14 high school teachers from eight culturally diverse urban schools attended a series of professional development workshops in statistics and completed a number of tasks to determine and develop their understanding of variability in a repeated sampling context. The Candy Bowl Task was used to probe teachers' notions of variability in such a context. Teachers' reasoning mainly revealed different types of thinking based on absolute frequencies, relative frequencies and on expectations of proportion and spread. Only one response showed distributional reasoning involving reasoning about centres as well as the variation around the centres. The conclusion was that a greater emphasis on variability and repeated sampling is necessary in statistics education in South African schools. To this end teachers should be supported to develop their own and learners' statistical reasoning skills in order to help prepare them adequately for citizenship in a knowledge-driven society.

Introduction

One just needs to open a newspaper, listen to the radio or watch television to realise that numbers and quantitative procedures are important in the world we live in. Every day we are flooded with statistics and have to rely on sensible quantitative reasoning to make decisions based on information in the media and in the workplace. These decisions concern most aspects of our lives and determine issues such as our health, prosperity, safety and much more; we have to be statistically literate to cope in our world. Being able to reason statistically

> empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world. (Steen, 2001, p. 2)

Statistics in most countries in the world, including South Africa, is not a separate subject in school curricula, but is included as a content area in the mathematics curriculum. Statistical thinking however differs from mathematical thinking. Cobb and Moore (1997) explain that this difference results from the focus in statistics on variability and the all-important role of context:

> Statistics requires a different kind of thinking, because data are not just numbers, they are numbers with a context. In mathematics, context obscures structure. In data analysis, context provides meaning. (p. 801)

Variability can be defined as the quality of an entity to vary, including under uncertainty. Bakker (2004a) elaborates on the relationship between variability and uncertainty: 'Uncertainty and variability are closely related: because there is variability, we live in uncertainty, and because not everything is determined or certain, there is variability' (p. 14). The interchangeable use of the terms variation and variability often causes confusion. Reading and Shaughnessy (2004) elucidate the difference between the two terms by defining variability as 'the [*varying*] characteristic of the entity' and variation as 'the describing or measuring of that characteristic' (p. 202). For instance, when learners write a test one can expect their marks to vary without being certain about how the marks will vary (the degree of dispersion of the marks). In order to describe the degree of dispersion, one needs some measure of how widely the marks vary in relation to, for example, the mean of the marks.

Variability is omnipresent in the world around us and affects all facets of our lives: 'It is variation that makes the results of actions unpredictable, that makes questions of cause and effect difficult to resolve, that makes it hard to uncover mechanisms' (Wild & Pfannkuch, 1999, p. 235). Variability is fundamental to statistical thinking and reasoning and the presence of variability in the world necessitated the development of statistical methods to make sense of data (Cobb & Moore, 1997; Franklin et al., 2005; Moore, 1990; Shaughnessy, 2007; Watson, 2006; Wild & Pfannkuch, 1999). It is self-evident that the development of statistical thinking and reasoning should be one of the major goals in statistics education and should take into account the omnipresence of variability in data (Franklin et al., 2005).

The South African school mathematics curriculum requires learners to be able to 'predict with reasons the relative frequency of the possible outcomes for a series of trials based on probability' (Department of Basic Education, 2011c, p. 36). Wessels and Nieuwoudt (2011) point out that the wording of the learning outcome and assessment standard quoted does not explicitly state the kind of skills needed for the statistical thinking and reasoning necessary in the statistical problem solving process. The Curriculum and Assessment Policy Statement (CAPS) documents for mathematics in the Senior Phase and Further Education and Training Phase do not use the words variation or variability in the context of data handling and probability (Department of Basic Education, 2011a, 2011b, 2011c), nor do they refer to the essence thereof in statistics. This is a serious shortcoming as variability is fundamental to and hence plays an all-important role in the statistical process (Franklin et al., 2005). Watson, Kelly, Callingham and Shaughnessy (2003, p. 1) state that 'statistics requires variation for its existence'. Data handling instruction without the inclusion of activities to develop learners' understanding of the role of variability is therefore questionable, and without doubt undesirable. A strong case could therefore be made for the pertinent inclusion of the fundamental statistical idea of variability, together with certain ideas of describing and measuring variation, in the South African mathematics curriculum.

Learners specifically need to develop the ability to reason about variability in samples and acquire a sense for expected variability to be able to predict results: 'When random selection is used, differences between samples will be due to chance. Understanding this chance variation is what leads to the predictability of results' (Franklin et al., 2005, p. 21). Exposure to appropriate activities to develop this understanding should therefore be frequent and well planned:

> [T]o improve students' feel for the expected variability in a sampling situation, students need considerable hands-on experience in first predicting the results of samples, and then drawing actual samples, graphing the results, comparing their predictions to the actual data, and discussing observed variability n the distribution. (Shaughnessy, Ciancetta & Canada, 2004, p. 184)

Research on teachers' statistical reasoning has up to now received little attention in South Africa (Wessels & Nieuwoudt, 2011). Many teachers lack a sound background in and proper understanding of statistics in general, and the ideas of variability and variance in particular; even if they did take statistics courses as part of their teacher education, such courses traditionally tend to be procedurally rather than conceptually inclined (Nieuwoudt & Golightly, 2006, p. 109), leaving many teachers lacking the critical proficiency to apply their statistical knowledge in practical settings (Wessels & Nieuwoudt, 2011). As a result of such 'incomplete' statistical learning experience, teachers do not feel confident about their own statistical reasoning (Shaughnessy, 2007) and lack pedagogical statistical knowledge. We concur that teachers have to understand how different fundamental statistical concepts are scaffolded in a child's mind to be able to prepare statistically literate learners who will be able to be critical consumers of the data they are deluged with every day (Wessels, 2009, p. 4). Opportunities should therefore purposely be created for teachers to develop their own reasoning about variability, and specifically about variability in sampling situations, to equip them better for the development of learners' awareness of variability present in samples or data in general.

Statistical reasoning in repeated sampling tasks

Data sets tell stories and these stories can usually be found in the variability in the data (Shaughnessy, 2007). Variability is not only present in the data, but occurs from one entire sample to another. Research on reasoning in repeated sampling situations mainly developed around tasks where repeated samples were taken from a known mix of differently coloured candies (sweets) in a bowl (Kelly & Watson, 2002; Reading & Shaughnessy, 2000; Shaughnessy et al., 2004; Shaughnessy, Watson, Moritz & Reading, 1999; Watson & Shaughnessy, 2004). An example of a task used to explore the way learners reason about the variability of data in a sampling context, is the Lollie Task. The Lollie Task or Candy Bowl Task (Figure 1), evolved from the Gumball Task, an item in the 1996 National Assessment of Educational Progress in the United States (Shaughnessy et al., 1999; Torok & Watson, 2000). In the task, learners are asked to predict how many reds would be in a handful of ten candies pulled from a bowl of candies of known mixed colours.

Learners then have to predict how many reds are likely to be pulled from the bowl if this experiment was repeated five times, each time returning the candies to the bowl and mixing them up. More important than the actual predictions is learners' reasoning about their predictions.

Different versions of the task had been administered to thousands of learners from Grade 3–12, mainly in the United States and Australia (Kelly & Watson, 2002; Reading & Shaughnessy, 2000; Shaughnessy et al., 2004; Shaughnessy, 2007; Torok & Watson, 2000).

Reading and Shaughnessy (2000, 2004) categorised learner responses in interview tasks according to measures of centre as high or low, and measures of spread as *wide*, *narrow* and *reasonable*. Some learners predicted, for the expected numbers of reds pulled from the mix of 20 yellow, 30 blue and 50

The Lollie Task

A bowl has 100 wrapped lollies in it. 20 are yellow, 50 are red, and 30 are blue. They are well mixed up in the bowl.

Jenny pulls out a handful of 10 lollies, counts the number of reds, and records it on the board. Then, Jenny puts the lollies back into the bowl, and mixes them all up again.

Four of Jenny's classmates, Jack, Julie, Jason, and Jerry do the same thing. One at a time they pull 10 lollies, count the reds, and write down the number of reds, and put the lollies back in the bowl and mix them up again.

What do you think? (List version)

1. I think the numbers of reds the students pulled were:

   __________ __________ __________

   __________ __________

   I think this because:

2. I think the list for the number of reds is most likely to be (circle one):

   A) 8,9,7,10,9　　D) 2,4,3,4,3

   B) 3,7,5,8,5　　E) 3,0,9,2,8

   C) 5,5,5,5,5

   I think this because: (Choice version)

3. I think the numbers of reds went from (a low of) ______ to a high of _______.

   I think this because: (Range version)

Source: Shaughnessy, J.M. (2007). Research on statistics learning and reasoning. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 957–1009). Charlotte, NC: Information Age Publishing

FIGURE 1: The Lollie Task.

red candies in the bowl, all low numbers (all below 5) or all high numbers (all above 5), whereas others predicted a wide (range ≥ 8) or a narrow (≤ 1) range of numbers. Learners tend to give more reasonable predictions if they get the chance to draw their own samples from the candy mix (Shaughnessy, 2007).

A 'demonstrated questionnaire' of the Candy Task was developed for use in South Africa with Grade 6, 8 and 10 learners (Reading, Wessels & Wessels, 2005).

Based on the System of Observed Learning Outcomes (SOLO) taxonomy (Biggs & Collis, 1982), a conceptual model for the categorisation of reasoning in repeated sampling tasks emerged in the research on variability in sampling situations (Canada, 2004; Kelly & Watson, 2002; Shaughnessy, 2007; Shaughnessy et al., 2004). The SOLO taxonomy is a neo-Piagetian framework for the analysis of the level of sophistication or complexity of a response on a specific task (Biggs & Collis, 1982, 1991). According to this model, learner responses in the Candy Bowl Task display four distinctive patterns of reasoning, following a progression from iconic, to additive, to proportional and finally to distributional reasoning (Shaughnessy, 2007). *Iconic* reasoning, such as relating personal stories and using physical circumstances, is usually evident in younger learners' responses (Kelly & Watson, 2002). Examples of iconic reasoning do not refer to the actual contents of the candy bowl or the proportions of the candy mix in it. Such responses might refer to luck: 'Maybe they are lucky and will get all the reds' or the physical act of pulling out the candies: 'They might get more reds because their hand could find them' (Shaughnessy, 2007). *Additive* responses are characterised by reasoning where no acknowledgement is given to the role of proportions in the mixture; reasoning is just about absolute numbers or frequencies of reds in the candy mix. *Implicit proportional* reasoning focuses on ratio, percentage or probability of reds whilst referring back to the original composition of the mixture. *Explicit proportional* reasoning involves reasoning about sample proportions, population proportions, probabilities or percentages. Finally, *distributional* responses give evidence of reasoning about centres as well as the variation around the centres. Shaughnessy et al. (2004) categorise responses of secondary school learners in repeated sampling tasks in a chance setting into only three broad groups: additive reasoning, explicit and implicit proportional reasoning, and distributional reasoning. These authors regard proportional reasoning as the cornerstone of statistical inference and call for more opportunities for learners to improve their proportional reasoning skills. They furthermore emphasise that

> the power of proportional reasoning in statistical situations needs to be identified much more explicitly in order for our students to evoke the connections of proportional thinking to statistical settings. (p. 4)

Professional development of in-service teachers in statistics

In the past decade, statistics started to play a more important role in the school mathematics curricula in many countries and necessitated professional development initiatives for teachers involved in the teaching of statistics. Well-designed professional development materials can serve more than one purpose: not only to support teachers' statistical content and pedagogical content knowledge but also to research how they understand fundamental statistical ideas (Shaughnessy, 2007, p. 998). Many of the professional development and curricular activities all over the world were focused on Grade 1–9 teachers and were also coupled with research projects on the statistical thinking and reasoning of mathematics teachers (Burrill, Franklin, Godbold & Young, 2003; Canada, 2004; Friel & Bright, 1998; Makar & Confrey, 2002; Shaughnessy, Barrett, Billstein, Kranendonk & Peck, 2004; Shaughnessy & Chance, 2005; Watson, 2006).

In South Africa however, professional development in statistics was mostly aimed at Grade 10–12 teachers (North & Scheiber, 2008; Zewotir & North, 2011; Wessels & Nieuwoudt, 2011). Grade 1–9 teachers however also need training and support as they have to prepare learners for Grade 10–12 mathematics and statistics. Little evidence can be found in the literature of professional development initiatives to promote understanding of fundamental ideas in statistics education for Grade 1–9 teachers or of research on their statistical thinking and reasoning about these ideas (Wessels, 2009; Wessels & Nieuwoudt, 2011). Embedded in the professional development initiatives in statistics should be opportunities for teachers to conduct statistical investigations themselves to develop their own statistical literacy as well as appropriate competent statistical thinking and reasoning skills (Shaughnessy, 2007). To facilitate the transfer of the knowledge acquired through their own engagement in such statistical investigations, teachers should be supported in the classroom by researchers, advisors and colleagues after such professional development.

Research design and methodology

Purpose of the research

The research described here constitutes part of a larger research project aimed at the improvement of statistics teaching in Grade 8 and 9 in the secondary school. The first phase of the project comprised the profiling of Grade 8–12 mathematics teachers with regard to their statistical knowledge for teaching to determine their professional development needs in statistics education. The teacher profiling information provided a basis from which a professional development intervention to broaden their knowledge of statistics and of the teaching of statistics could be designed (Wessels & Nieuwoudt, 2011). Of particular interest in Phase 2, the intervention, is teachers' reasoning in a repeated sampling context.

Research approach

Throughout the project, investigations of an explorative nature were undertaken from an interpretivist view with the aim to understand what and how teachers made sense of their learning experiences. For the first part of the project a qualitative–quantitative multi-method research design was used (Wessels & Nieuwoudt, 2011); a qualitative design was employed for the second part of the study.

Research context

Building on research described in the literature (Burrill et al., 2003; Canada, 2004; Makar & Confrey, 2002; Shaughnessy et al., 2004; Shaughnessy & Chance, 2005; Watson, 1998, 2006) and the analysis of the profiling questionnaire (Wessels & Nieuwoudt, 2011) used in the first part of the project, a series of eight professional development workshops in statistics were developed. The two main sources used in the developing process were a sequence of modules for pre-service teacher training in statistics in the United States developed by Canada (2006) and a professional development programme in statistics for in-service teachers in Australia developed by Watson (1998).

A problem-centred approach was used as point of departure for the series of workshops, focusing on *statistical knowledge for teaching*, which consists of content knowledge and pedagogical content knowledge in statistics. This model of statistical knowledge for teaching is based on the construct of *mathematics knowledge for teaching* developed by Ball, Thames and Phelps (2008), and includes:

- knowledge of statistics content and of relationships between statistical ideas (specialised content knowledge)
- knowledge of how students understand statistics concepts and develop statistical thinking and reasoning (knowledge of content and students)
- knowledge of how the statistics content should be facilitated and taught (knowledge of content and teaching)
- knowledge and interpretation of the curriculum
- knowledge of the use of technology to develop statistical thinking and reasoning (technological pedagogical content knowledge)
- the development of competent statistical thinking and reasoning skills of teachers.

Seven of the workshop sessions lasted two hours each; a four-hour session on the use of the computer data exploration software Tinkerplots® (Konold & Miller, 2005) in a computer laboratory was also included. The first six workshops were presented twice a week, one on a weekday with a repeat on Saturdays for teachers who could not attend during the week because of full schedules. During the workshops teachers' statistical knowledge as well as statistical thinking and reasoning skills were developed through rich learning experiences that included all components of the statistical investigation process: posing problems, collecting data, analysing them, drawing conclusions and making predictions. The following topics were addressed in the workshops (Wessels, 2009):

- Data generation, representation and analysis of single categorical data sets to develop the language, argumentation skills and mindset for exploratory data

analysis as preparation for the comparison of multiple data sets later in the programme.
- The influence of variability in measurement of a single person on interpretation of findings for the whole class.
- Identifying trends in data amidst variability.
- Chance as a precursor to probability.
- Repeated sampling from a known and then an unknown population.
- Probability.
- Using the Internet and statistics education software in the development of statistical thinking and reasoning.
- Informal inferential reasoning – comparing data sets of the same and different sizes.

Participants

The research was conducted in a large city in the Gauteng province of South Africa in 23 socio-economically and culturally diverse schools (Wessels & Nieuwoudt, 2011). Mathematics teachers from all 23 schools where the profiling questionnaires were distributed were invited to attend the series of eight workshops. Fourteen Grade 8–12 teachers (13 women and one man) from eight schools regularly attended the series of workshops. The target group for the professional development workshops initially was Grade 8 and 9 teachers, but a number of Grade 10−12 teachers requested to be included. This article describes and explores these Grade 8–12 teachers' reasoning on one of the tasks they completed at the beginning of the workshop on repeated sampling. None of the teachers had had any previous exposure to repeated sampling in a probability setting.

Most of the previous research on variability in a repeated sampling context dealt with the statistical reasoning of students and pre-service teachers in such situations (Canada, 2004; Reading et al., 2005; Shaughnessy et al., 2004; Torok & Watson, 2000; Kelly & Watson, 2002). The study described in this article focused on variability reasoning in a repeated sampling context of a different population, namely of in-service teachers.

Data generation and analysis

To answer the research question of how teachers reason in a repeated sampling context, participating teachers engaged in a set of four tasks about probability distributions developed by Canada (2004). These tasks were set midway through the series of professional development workshops, before the two workshops focusing on repeated sampling in a probability context. The version of the Candy Bowl Task comprised three questions shown in Figure 2 (Zawojewski & Shaughnessy, 2000, p. 259).

Following Shaughnessy (2007), the model of four distinctive types of reasoning according to level of sophistication or complexity of a response was then used as point of departure to analyse the teachers' responses to the questions. According to this model, participant responses in the Candy Bowl Task were expected to display iconic, additive, proportional or distributional thinking in repeated sampling tasks. Corresponding codes and code descriptors developed and refined by Reading and Shaughnessy (2000) and Shaughnessy et al. (2004) were used in the analysis of the data.

Trustworthiness

Validity of the Candy Bowl Task as an instrument was established in earlier research studies (Kelly & Watson, 2002; Reading & Shaughnessy, 2000; Shaughnessy et al., 2004; Shaughnessy, 2007; Torok & Watson, 2000). For the purpose of Phase 2 of the study project reported here, coding reliability was established by double coding of responses by an independent researcher with experience of the analysis of responses in the Candy Bowl Task. The two coders had a 95% agreement on the coding of all responses and, after discussion, consensus was reached on the other 5%.

Ethical considerations

Approval to conduct the research was obtained from the tertiary institution where the project was hosted. Information about the project was shared with all schools and participating teachers. Teachers were invited to participate in the workshops; therefore participation was voluntary. Teachers could at any time end their participation in the intervention. The researcher communicated the commitment to all participants and principals to keep results of the research confidential and report on the research and findings anonymously. Data, results and findings have strictly been used for the stated research purpose only.

Findings and discussion

Findings for each part of the Candy Bowl Task are discussed in turn, supported by examples from teachers' responses to illustrate aspects of their understanding of variation.

This set of questions helps to give a picture of how you currently are thinking about some problems in probability and statistics. Rather than think in terms of a right or wrong answer, just write down your best thinking for each situation. The most important thing you can do is communicate your reasoning.

Suppose there is a container with 100 pieces of candy in it. Sixty are red, and 40 are yellow. The candies are all mixed up in the container. You reach in and pull out a handful of 10 candies at random.

1. How many red candies do you think you might get? Why do you think this?
2. Suppose you do this several times (each time returning the previous handful of 10 candies and remixing the container). Do you think this many reds would come out every time? Why do you think this?
3. Suppose six classmates do this experiment (each time returning the previous handful of 10 candies and remixing the container). Write down the number of reds that you think each classmate might get. Why did you choose those numbers?

Source: Zawojewski, J., & Shaughnessy, J. (2000). Data and chance. In E. Silver, & P. Kenney (Eds.), *Results from the seventh mathematics assessment of the National Assessment of Educational Progress* (pp. 235–268). Reston, VA: National Council of Teachers of Mathematics

FIGURE 2: The Candy Bowl Task.

Analysis of Candy Bowl Task questions

Question 1

First part: 'How many red candies do you think you might get?'

This question aimed to determine whether teachers expected variation when taking a sample. In this task two teachers expected something other than six candies in the handful of candies drawn from the bowl, with responses 'any number, more red than yellow' and '± more than half more than yellow'. Three teachers acknowledged variability and indicated a range rather than one number of candies with responses such as '4–6' and '0 ≤ red ≤ 10'. More than half of the teachers (57%) however answered that they expected six red candies to be drawn (Table 1). This result concurs with other research findings on learner and pre-service teachers' reasoning about variability in the Candy Bowl Task (Shaughnessy et al., 2004; Watson & Shaughnessy, 2004). A probable reason for this focus on centre and not possible variability could be that the teachers' experiences with and understanding of theoretical probability in their mathematics and statistics education or in professional development fostered limiting constructions and hindered their understanding of variability in a sampling context (Shaughnessy et al., 2004). Their understanding of what mathematics is could also have prompted a single value point answer, not considering possible variability present in repeated samples. They might even have considered possible variability but felt that they had to give a single point answer.

Second part: 'Why do you think this?'

In this second part reasons for expectations were teased out. The responses were categorised with codes and code descriptors distinguishing between iconic, additive, proportional and distributional reasoning (Shaughnessy et al., 2004). Results are summarised in Table 2 and discussions are elucidated with examples of participants' reasoning.

Iconic reasoning: No reason or a vague reason is given, or participant refers to physical circumstances. Three teachers responded with iconic reasoning: 'The candies are all mixed, you put your hand in and take any 10 candies. You cannot feel whether they are red or yellow' and 'Can't be more than 10 or less than 10'.

Additive reasoning focuses on frequencies and not relative frequencies. No acknowledgement is given to the role of proportions in the mixture; reasoning is just about absolute numbers or frequencies of reds in the candy mix. Three teachers responded with additive reasoning, for example '20 more red candies than yellows' and 'possibility of getting more reds than yellows'.

Implicit proportional reasoning attends to ratio, percentage or probability of reds whilst referring back to the original composition of the mixture. One teacher's response showed implicit proportional reasoning, referring to the composition of the mixture with a fraction: 'Will be $\frac{2}{3}$ red, that is the grouping in the container'. It is incorrect to say that $\frac{2}{3}$ are red, but the proportion of yellow to red candies is 40:60, which relates to 2:3 and might have prompted this assumption. This comment could however be indicative of centre as $\frac{2}{3}$ is more than half.

Explicit proportional reasoning involves explicit reasoning about sample proportions, population proportions, probabilities or percentages. Six teachers used explicit proportional reasoning for this question: '60% is red candies and 40% is yellow' and 'Ratio for red and yellow is 6:4'.

Distributional reasoning involves reasoning about centres as well as the variation around the centres. None of the teachers used distributional reasoning. An example of distributional reasoning is: 'The number of red candies will be 6, but also spread out around 6.'

In summary: six out of the 14 participants responded to Question 1 with *iconic reasoning* or *additive reasoning*. Seven teachers employed *implicit proportional reasoning* or *explicit proportional reasoning*. Only three considered a range of possible outcomes for a handful of ten candies drawn from the bowl.

Of the latter three teachers, one backed up her expected number of reds (4–6) with *iconic reasoning* ('The candies are all mixed, you put your hand in and take any 10 candies. You cannot feel whether they are red or yellow'). The second teacher's expectation for the number of red pulled from the bowl was '10 ≤ red ≥ 10'; she supported her choice with iconic reasoning: 'Can't be more than 10 or less than 10'. It can be argued that this teacher was referring to a range, but because of teachers' busy schedules they were not available for individual interviews and therefore responses had to be analysed as presented. The third teacher also expected a range, answering '±6', but used *explicit proportional reasoning*: 'Ratio 100:10 and $\frac{6}{10}$ red, $\frac{4}{10}$ yellow'.

TABLE 1: Responses to Question 1: Expected number of reds.

Response	Number (N = 14)
No answer	1
Other than 6 or a range	2
6 red	8
A range	3

TABLE 2: Summary of responses to Question 1: Reasons for expected number of reds.

Type of reasoning	Number (N = 14)
No answer	1
Iconic reasoning	3
Additive reasoning	3
Implicit proportional reasoning	1
Explicit proportional reasoning	6
Distributional reasoning	0

None of the teachers used *distributional reasoning*, showing that their ability to reason about variability in a repeated sampling context may not be well developed. One of the reasons for this situation may be a lack of exposure to variability in repeated sampling situations before the intervention alluded to in the group interview. Too much emphasis on a procedural rather than a conceptual approach when engaging with the ideas in previous training could also be at the root of the observed phenomenon.

Question 2

Suppose you do this several times (each time returning the previous handful of 10 candies and remixing the container). Do you think this many reds would come out every time? Why do you think this?

Question 2 aimed to determine whether teachers *expected* variability in samples. Responses to Question 2 were analysed according to the codes and descriptors developed by Shaughnessy et al. (2004) and are summarised in Table 3.

One teacher did not respond to this question. Two teachers were of the opinion that the same number would come out every time – their responses were categorised on level 0.

Ten of the 13 teachers indicated that the number of reds pulled out will not be the same every time. These responses however were on different levels because of the reasons given. Of these 'no' answers, eight showed level 1 reasoning: 'Just a chance', and 'We have red and yellow candies – chance that there is many yellow or red'. One teacher's answer gave implicit indication of variation with no further specific mention of the distribution and was categorised as a level 2 response: 'Not a chance that you will draw the same number of reds every time'.

One teacher's response was considered level 3. She answered that the number of reds pulled would be 'more or less' the same each time and motivated her answer with 'ratio 6:4'. One of the teachers responded that the number of reds would not be same every time, her reason being: 'Random, can be red or yellow. Average colours will indeed be more red than yellow'. The reasoning does not explicitly use proportions but may be indicative of a consideration of variability and uncertainty and might be considered an intermediary stage to proportional thinking.

Question 3

Suppose that six classmates do this experiment, each time returning the previous handful of 10 candies and remixing the container. Write down the number of reds that you think each classmate might get. Why did you choose those numbers?

This question aimed to elicit teachers' predictions for the results of repeated samples and responses were analysed using a four-point rubric as shown in Table 4 (Shaughnessy et al., 2004).

Of the inappropriate predictions, none was *narrow* or *high*, whereas two each were *wide* or *low*. Eight teachers gave a *reasonable* prediction for the number of red candies to be pulled in repeated trials. *Reasonable* predictions are spread in a more normative way around the centre, such as 2, 4, 6, 5, 8, 7.

The results in Table 5 show that half of the teachers giving an appropriate range used additive reasoning whilst the other half displayed proportional and distributional reasoning. Conversely, all teachers using proportional and distributional reasoning suggested a reasonable range for the number of reds pulled from the mix. This result concurs with the research of Watson and Shaughnessy (2004) who point out that in their research participants using explicit proportional reasoning 'were more likely to suggest a reasonable amount of variation around the expected mean of the samples' (p. 108).

In a group interview after completing the task, teachers admitted that they had not been exposed to any activities

TABLE 3: Codes for Question 2: Reasons for expectations in repeated trials.

Answer	Reasoning Level
Yes	**0 – Yes** If the answer was 'Yes', but their reasoning indicated they knew things would vary, responses were coded according to the 'No' coding scheme below.
No	**1 – Iconic/Additive reasoning** No reason given; vague or nonsense reason; 'could be anything' reasoning; additive reasoning such as 'there are more reds'.
	2 – Implicit proportional reasoning Some implicit indication of variation – 'around 6' – but no explicit information about the distribution or about proportional reasoning, for example, 'won't be the same every time', 'probability is not the same every time'.
	3 – Explicit proportional reasoning Explicitly mentioning the ratio, percentage, or chance of reds (60% reds, 6:4 ratio) or reasonable spread; some clear indication was given of proportional reasoning about the distribution of outcomes.
	4 – Distributional reasoning Explicit use of a reasonable spread, as well as a spread around the expected value.

TABLE 4: Codes for Question 3: Reasons for expectations.

Coding	Description
0	Too much or too little variation, for example, *narrow*: range ≤ 1; *wide*: range ≥ 8; *low*: all ≤ 6; *high*: all ≥ 6
1	Appropriate range of choices, but inappropriate or additive reasoning
2	Using ratio/average/chance/spread – some indication of proportional reasoning
3	Explicitly using variation combined with centres demonstrating distributional reasoning

on variation in a repeated sampling context prior to the professional development workshops. In summary, analysis of teacher responses shows that although only one teacher employed distributional reasoning in one of the tasks, varying levels of reasoning – from iconic, to additive, to proportional reasoning – were found across the different tasks.

To assess if teachers had a better understanding of variability after the intervention and were able to apply this understanding to related tasks, several tasks were given to them two weeks after the two sessions on variability. These tasks (called post intervention tasks) included questions concerning a 50:50 (black:white) spinner, the tossing of a die, the wait times at two movie theatres and the comparison spelling test scores of two classes. Results of only the first three questions about the spinner will be presented here because the questions concerning the spinner were similar to those in the Candy Bowl Task, except that they focused on probability. Instead of sampling from a mix of candies, teachers this time had to predict the number of times a 50:50 (black:white) spinner would land on black in 50 spins and six sets of 50 spins. The task, used by Canada (2004) in his research with pre-service teachers as participants, consisted of three questions (Figure 3).

Only eight of the original 14 teachers who participated in the Candy Bowl Task attended the session in which the Spinner Task was completed.

Analysis of post intervention questions

Question 1

Question 1 concerned the teachers' expectations for the number of times the arrow would land on black. Three teachers displayed iconic reasoning: 'Not sure', backing the answer up with 'It can be any number'. Five teachers showed explicit proportional reasoning: 'Between 10 and 40 times' because 'the probability is 50%'. Another teacher showing explicit proportional reasoning answered '40–60% of the times', stating that physical causes such as not lining up the spinner in the same spot for each spin, or not applying the same amount of force, would account for the variation,

Question 2

This question focused on the comparison between two samples of 50 spins. Three teachers said that the results of the second set of 50 spins would be more or less the same as the first set, whereas five teachers implicitly or explicitly referred to possible variation between the two sets.

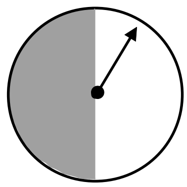
Question 3

Question 3 required a list of and a motivation for the expected outcomes for six sets of 50 spins. One teacher misunderstood the question and gave the expected outcomes for six spins with an unreasonable spread (from 1 to 6). Four teachers gave a reasonable spread for the expected number of spins (between 20 and 30), one gave a wide spread

TABLE 5: Summary of responses to Question 3: 'Write down the number of reds that you think each classmate might get. Why did you choose these numbers?'

Coding	No. of responses $N = 14$	Examples of teacher responses
0	6	One teacher gave the expected number of reds: (2, 2, 3, 4, 6, 7) with no reason. Another teacher gave her choice of expected number of reds (1, 2, 4, 5, 6, 7) but added that she guessed. Two teachers gave *low* responses indicating too little variation: (2, 3, 4, 5, 5, 6) and (2, 3, 4, 4, 5, 6). Two teachers gave *wide* responses: (1, 0, 2, 5, 6, 10), whereas the other did not give actual numbers, just said 'Each will draw a different number of each colour. Possible between 0 and 10'.
1	4	The teachers gave an appropriate range, but backed up their choices by *additive reasoning*, for example, 'Red is more' or 'Reds in majority, but possibility indeed there for less [*sic*]'.
2	3	Teachers' reasoning indicated an *appreciation for proportion* in motivating their choices with an appropriate range, for example, 'Average will be 6' and 'Chances are 6 out of 10, but can be 10 also'.
3	1	This teacher employed *distributional reasoning*, explicitly using variation combined with centre, adding to a reasonable range (4, 4, 5, 5, 6, 7): 'Numbers that are so near to the average but also spread out a little'.

Consider the spinner on the right:



1. Matt is curious to see how often the spinner lands on black, so he spins it 50 times. How many times (out of 50 tries) do you think the arrow might land on black?
2. Why do you think this?
3. After Matt's first set of 50 spins, he decides to do a second set of 50 spins. How do you think his results on the second set of 50 spins will compare with the results of his first set?
4. Matt actually has a lot of time on his hands, so the next day he does six sets of 50 spins.
5. Write a list that would describe what you think might happen for the number of spins out of 50 the spinner would land on the shaded part in each of the six sets of 50 spins.

___ (Out of 50) ___ (Out of 50)

___ (Out of 50) ___ (Out of 50)

___ (Out of 50) ___ (Out of 50)

Why did you choose those numbers?

Source: Canada, D. (2004). *Preservice elementary teachers' conceptions of variability*. Unpublished doctoral dissertation. Portland State University, Portland, OR, United States

FIGURE 3: The Spinner Task.

(between 15 and 35) whereas another one gave an improbably large spread of 18–45.

The teacher who misunderstood the question gave an incomplete reason for her expectations. Of the other seven, one teacher displayed explicit proportional reasoning, bordering on distributional reasoning: 'Big variation where it will land. Therefore I have made my choice between 40–60% of the total'. The other six teachers all showed distributional reasoning: 'It is more or less half of 50' and 'It gives me the 50% chance, but with some variation'.

These results of the post intervention questions clearly show an increased awareness of variation in multiple trials and a shift from iconic reasoning to proportional and distributional reasoning after the intervention on variability.

The results of this study point to teachers' lack of familiarity with and understanding of variability in a repeated sampling context before the intervention. Analysis of the post intervention questions shows that teachers' understanding of the concept of variability grew during the intervention and that 75% of the eight teachers were able to transfer this increased understanding to related tasks.

Implications for teaching

A number of crucial statistics concepts are under-emphasised in the South African mathematics curriculum (Wessels & Nieuwoudt, 2011). Bakker (2004a, p. 273) states that 'the most fundamental key concepts in statistics are variability and uncertainty' and points out that a sense of the variability present in a certain event creates the need to consider a sample or a distribution of the data. Yet, the words 'variability' and 'uncertainty' are not mentioned in the CAPS for mathematics in Grade 10–12 even once (Department of Basic Education, 2011b, 2011c). Appropriate learning experiences, aimed at facilitating teachers' profound understanding of the mentioned and other fundamental statistical ideas, need to be developed to afford teachers the opportunity to engage meaningfully and purposely with the ideas in ways that will support them in their decision making regarding the teaching and learning of the ideas in their classes.

Reasoning about centre and spread in a data set needs to be scaffolded carefully through well-chosen tasks and rich discussions, supported by a classroom culture conducive to independent thinking and shared understanding. Although reasoning about repeated trials in the face of probability is mentioned in the curriculum (Department of Basic Education, 2011c, p. 36), no mention is made of the crucial underlying concept of variability in repeated sampling. Another crucial aspect, proportional reasoning, is regarded as the 'cornerstone of statistical inference' (Shaughnessy et al., 2004, p. 184) and connects statistical reasoning in data and probability (Watson & Shaughnessy, 2004). Proportional reasoning is likewise not mentioned in connection with statistics in the curriculum.

The development of learners' notions of these important concepts is dependent on teachers' in-depth and in-breadth knowledge and well-developed thinking and reasoning skills about these concepts. It is therefore crucial that teachers' awareness and understanding of variability and uncertainty in specific situations, such as repeated sampling and probability situations, is developed through well-planned activities and discussion opportunities that can be provided during professional development experiences. Teachers find the development of reasoning about variability in samples and distributions challenging (Bakker, 2004b) and might avoid teaching it.

Most teachers who participated in the study had experienced a traditional education in statistics, emphasising procedural rather than conceptual competence, and had typically not been exposed to activities that could build their understanding of and reasoning about repeated sampling (Wessels & Nieuwoudt, 2011). Research about variability in a repeated sampling environment that included an intervention (Canada, 2006) has shown an improvement of participants' descriptions of what they expected (*description*) as well as of reasons for their expectations (*causality*). After this intervention, participants increasingly appreciated how variation occurred in multiple trials whilst reasons for their expectations improved, and progressively emphasised proportional reasoning coupled with a realisation of what is likely in the presence of variation. The results of our study concur with the results reported by Canada. Canada emphasises that to be effective, teacher education programs need to include an environment where teachers 'can learn in a similar way that they themselves will aim to teach' (p. 61). Teachers must get the opportunity to draw real samples and discuss differences and similarities in the distributions to develop their own skills about this topic so as to enable them to facilitate the development of proportional reasoning skills and a sense for expected variability in learners. The value of proportional thinking in repeated sampling situations also needs to be made explicit (Shaughnessy et al., 2004; Watson & Shaughnessy, 2004).

Limitations of the study

The research question of this study focused on teachers' reasoning in a repeated sampling context. The sample was limited by the fact that only 14 volunteering teachers participated. A larger sample would have yielded a better picture of teachers' reasoning with repeated trials. The focus of the article however is not to generalise, but to summarise what can be learnt from the responses of these teachers.

The fact that teachers' time to participate was limited made individual interviews impossible – a fact that curtailed conclusions from their responses. Reasons for their responses could not be probed in depth; for example, in Question 1 of the Candy Bowl Task, one teacher gave the range '1, 2, 4, 5, 6, 7', leaving out 3 as a possible number of reds to be pulled from the candy bowl. Did she intentionally break the sequence to impart that chances are that some numbers will not be drawn? She responded with a range (4–6) to Question 3, backed up by *iconic reasoning*, responded with explicit proportional reasoning on Question 2 and then said she guessed the six possible numbers for reds in successive trials. It is only possible to determine the realistic level of her reasoning through an in-depth interview. The question can also be asked about what notions of proportionality teachers held at the beginning of this research project.

The importance of variability as a fundamental concept in statistics necessitates more research about teachers' and learners' reasoning about variability, and especially their reasoning about variability in a repeated sampling context. Interview tasks to probe participants' thinking more thoroughly should be an integral part of such a study.

Conclusion

Shaughnessy (2007) points out that beliefs and conceptions about outcomes of repeated trials do not easily change, and emphasises the importance of including empirical experiments and simulations in data handling and probability instruction. It is crucial to be involved in such activities on a regular basis. The following citation is just as true for teachers as for students:

Beliefs and conceptions about data and chance are very difficult to change, and research has suggested that empirical experiments and simulations must be systematically built into instruction over a longer period of time in order to change the patterns of students' intuitive conceptions. (p. 976)

A greater emphasis on variability and repeated sampling is necessary in statistics education in South African schools to develop learners' statistical reasoning skills and prepare them adequately for citizenship in our knowledge-driven society. Pre-service teacher education programs as well as professional development experiences in statistics of in-service teachers need to include ample opportunities for developing competence with regard to proportional thinking and an appreciation of the role of variability and uncertainty in statistics in order to equip them for this task. To this end, purposeful and effective opportunities should be created for teachers to engage collaboratively with activities and relevant materials to gain such competence and statistics teaching proficiency.

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Competing interests

The authors declare that they have no financial or personal relationship(s) that may have inappropriately influenced them in writing this article.

Authors' contributions

H.W. (University of Stellenbosch/North-West University at time of the research) initiated and conducted the research and drafted the manuscript. H.N. (North-West University) contributed to the conceptualisation, planning and execution of the research, as well as the analysis of the data and the finalisation of the manuscript.

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