The effect of a group approach on the performance of high school mathematics learners

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In this study, we investigated the effects of a group approach versus a non-group approach on the mathematics performance of learners. A group approach refers to an arrangement in which learners sit together to discuss and solve mathematics tasks. We studied a convenience sample of low-performing Grade 10 mathematics learners using a quasi-experimental design with a non-equivalent control group. The experimental group was taught using a group approach and the control group using non-group approach instruction. To measure the effects of teaching approaches, we administered a Financial Mathematics Achievement Test (FMAT) before and after the experiment. Using a one-way analysis of covariance (ANCOVA) the study found that learners in a group approach learning environment performed significantly better than learners taught through a non-group teaching mode ($p < 0.05$). The theory of cognitive load was used to interpret the results. The results suggest that a group approach may be effective when teaching certain mathematics topics in Grade 10 classrooms.

Introduction

We often hear people say 'many hands make work light', implying that two heads are better than one. This adage speaks well of the potential of a group approach in helping individuals to accomplish more work than can be achieved in solitary pursuits. In essence, when people work in groups they can share responsibilities and ideas. Consequently they may be more successful in finding a solution to a problem. Within the context of a modern discourse the term *group work* or *group approach* is used interchangeably with the term *collaboration* (Barnes, 2003; Edwards & Jones, 2001). In various spheres of inquiry people are beginning to appreciate the beneficial influence of utilising collaborative initiatives to foster productive interdisciplinary approaches. Within the research paradigm collaboration is seen as a useful tool to become familiar with the many resources available in the facilitation of research processes, practice and partnership (Erichsen, Goldenstein & Kaiser, 2011). Even though some researchers have emphasised the difference between the terms *collaborative group learning* (a specific approach to group work that emphasises the importance of roles) and a *group approach* (a more general term), for the purpose of the discussion in this article, both terms will be used interchangeably (Barnes, 2003; Davidson & Kroll, 1991; Edwards & Jones, 2001; Lai, 2011; Yackel, Cobb & Wood, 1991). In this study, the terms collaboration and group approach refer to an arrangement in which two or more people work together to achieve a common goal. In this arrangement strategies are integrated in an attempt to address problems, topics or issues of a complex nature (Erichsenet al., 2011)

Given this background, it seems reasonable to propose that schools should also see the need to train learners to become effective in collaborative learning settings. Therefore, instruction that promotes collaborative skills of learners ought to be designed. In this article a collaborative or group approach refers to a classroom arrangement in which learners sit together to discuss and solve mathematics tasks or problems (see Dhlamini, 2012). Our premise is that a classroom arrangement that incorporates group learning activities provides learners with 'effective tools to reinforce their problem solving system' (Dhlamini, 2012, p. 241). This is possible because the processes that occur during group discussion include verbalising explanations, justifications and reflections (Beers, Boshuizen & Kirschner, 2007; Kirschner, Beers, Boshuizen & Gijselaers, 2008), giving mutual support (Van Boxtel, Van der Linden & Kanselaar, 2000) and developing arguments about complex problems (Munneke, Andriessen, Kanselaar & Kirschner, 2007). In the same vein, Dhlamini (2012) emphasises three elements of group learning activities: discussion, argumentation and reflection. According to Van Boxtel et al. (2000), group learning activities can allow learners to provide explanations of their understanding, which can help them to elaborate and reorganise their knowledge. Lai (2011) notes that group learning activities, such as providing elaborated explanations to group members, improve learner comprehension of conceptual knowledge.

In this study, we constructed *group approach* learning environments in experimental schools. Control schools mostly followed a *didactic teaching approach* in which teaching is associated with transmission of knowledge by the teacher, and learning associated with passive receiving of knowledge. The didactic teaching approach primarily involves lecturing and is essentially teacher-centred, limiting learner participation and reflection (Johnes, 2006). This approach is usually associated with traditional approaches to teaching. In almost all control schools teachers employed traditional methods of teaching that were primarily presented in a non-group teaching mode. Therefore, this study investigated the effects of the two teaching approaches on the performance of Grade 10 mathematics learners. Our primary research question was: What is the effect of using a group approach on the performance of learners when teaching certain topics in Grade 10 financial mathematics?

In light of the national search for teaching approaches that can improve mathematics achievement of learners in South Africa, we believe that our outcome variable, Grade 10 performance in financial mathematics, is particularly timely. Indeed, our results may be of importance for those interested in empowering teachers to meet the challenges of the new curriculum.

Theoretical framework

A group approach may be established within the broader theoretical framework of *cognitive load*. Cognitive load refers to the mental burden and effort that an individual endures whilst executing a problem solving task (Chen, 2003), and is largely linked to a *working memory*, which is considered to be influential in determining the success of learning. Working memory or the 'short-term memory' is the part of the memory, or human cognitive architecture, that is needed to process incoming information (Kirschner, 2002). However, the limitations of the working memory in processing ability and duration are well documented and widely accepted within cognitive science research (Dhlamini & Mogari, 2011). Concerning its processing duration, researchers such as Paas, Van Gog and Sweller (2010) argue that almost all information stored in working memory and not rehearsed is lost within 30 seconds. Also, the working memory's capacity cannot deal with more than about seven elements of information simultaneously (Engle, 2010). Hence, if the working memory capacity is exceeded whilst processing information then some or all information may be lost. When the working memory is unable to deal with or process information, the cognitive load may be said to be too high.

In contrast, long-term memory has the capacity to permanently store huge amounts of knowledge and chunks of domain-specific skills in a form of cognitive schemas or schemata (Dhlamini & Mogari, 2011). A schema can hold a huge amount of information which can be treated as a single unit when it is processed in the working memory (Kirschner, 2002). Given that a schema can be treated as a unit, it may be processed effectively in the working memory. Cognitive load is reduced when information is treated as a unit in the working memory. For this reason cognitive load theory has focused on designing teaching methods to counter the effect of the limitations of human working memory which, when not managed properly, may increase cognitive load (for examples, see Dhlamini, 2012; Dhlamini & Mogari, 2012a).

Although cognitive load theory generated lessons are meant to manage individual working memory load (cognitive load) of individuals (Kirschner, Paas, Kirschner & Janssen, 2011), Kirschner, Paas and Kirschner (2009) have emphasised an alternative technique of effectively dealing with individual working memory limitations by making use of the multiple working memories of individuals in group approach learning environments. From a cognitive load theory perspective, it is argued that dividing the processing of information in the working memory across individuals in a group approach environment is useful because this technique allows information to be divided across a larger reservoir of cognitive capacity, thus increasing the working memory capacity (Kirschner et al., 2011). According to Dhlamini (2012), in a group approach learning environment, which represents a huge working memory system, the limitations of individual working memories are not exposed because individual working memories are not subjected to processing each piece of problem-solving information. Therefore the risk of overloading each group member is lowered, and an individual's working memory capacity is freed up whilst the group's collective working memory capacity is expanded, and the cognitive load may be reduced (Kirschner, 2009).

According to Kirschner (2009), within a group setting information processing is characterised by active and conscious sharing (i.e. retrieving and explicating information), discussing (i.e. encoding and elaborating the information) and remembering (i.e. personalising and storing the information) valuable task-relevant information and knowledge held by each group member. For a group to perform a mathematics task, it is not necessary that all group members be highly knowledgeable in task-related information or be able to process all available information by themselves and at the same time (Johnson, Johnson & Stanne, 2001). As long as there is communication and coordination between group members, the information elements within the task and the associated cognitive load can be shared amongst group members (Kirschner, 2009).

Given that there is a paucity of studies to test whether or not learners in group approach settings perform better than learners in non-group approach settings (Kirschner et al., 2009), we decided to investigate this observation. The aim of this study therefore was to investigate the comparative effects of a group approach and a non-group approach on the performance of Grade 10 mathematics learners. In a sense, this study was conducted within the 'effect paradigm'. Studies conducted within the effect paradigm examine outcomes of collaboration rather than the collaborative process itself. In effect the comparison is between a group performance and a non-group or individual performance (Lai, 2011). Some of the results from these studies suggest that a group approach can have powerful effects on learner performance (Lai, 2011).

Methods

Study design

Our study is located within an eclectic paradigm, meaning that we did not follow a single research methodology to conduct our investigation. Both quantitative and qualitative methods were used to collect data. Although we conducted a quasi-experimental study (classroom-based) with a non-equivalent control group, we included aspects of a descriptive survey design to determine whether or not qualitative results will account for the quantitative results of the quasi-experimental study. Random allocation of participants to either an experimental group or a control group was not possible due to practical constraints. For example, we wanted to assign individual learners from all the schools into each group. This, however, was not possible because the principals would not accept the possible disruption this could cause in their schools. To prevent contact of two groups, experimental schools and control schools were chosen such that they were separated by a distance of about 80 kilometres. According to Gaigher (2006), such separation effectively prevents diffusion, contamination, rivalry and demoralisation.

Participants

Participants for the study consisted of a convenience sample (N = 783) of Grade 10 mathematics learners from nine township high schools. Of the 783 learners, 413 from five schools formed the experimental group and 370 from four schools formed the control group. As a criterion, the selection of schools was based on their Grade 12 performance indicators of the year that preceded the study. Based on these indicators we projected the performance status of learners at the beginning of the study. Prior to the commencement of the study, we collected the data in Table 2 to verify the suitability of schools to participate in the study.

Ethical considerations

Ethical clearance for the study was obtained through our institutional ethics committee and schools involved consented to their participation in the study. In addition, all learners gave informed consent that permitted us to use the collected data in the study.

Procedure

Learners in the control group were taught by their four teachers under conventional conditions; the first author implemented a group approach in all five experimental schools. The reason for the avoidance of a teacher component in experimental schools was to eliminate variations in the implementation of a group approach. In addition, teachers would have required training on how to use a group approach, and this might have prolonged the study. Even though it may be argued that a stranger, in the form of the first author, was introduced in experimental schools thus creating an artificial and unusual learning atmosphere, the assertion is that this arrangement ensured that a group approach was uniformly and appropriately administered. One of the requirements of conducting experimental research is that the 'groups that are to receive the different treatments should be equated on all variables that may influence performance on the dependent variable' (Gay, Mills & Airasian, 2011, p. 252).

The intervention in each school lasted for two weeks. During the intervention the pairing of schools was such that each experimental school was paired with a control school. Whilst implementing a group approach in experimental schools the first author visited each control school once to observe a comparative mathematics lesson being implemented (see Figure 1). Control schools were visited on days on which the first author had little teaching time at corresponding experimental schools. The fifth experimental school was not paired with a control school, but this did not affect the results because results in each group were aggregated.

Instruments

The principal instrument for data collection was a standardised Grade 10 Financial Mathematics Achievement Test (FMAT). Test items were sampled from previous Grade 10 examination question papers, and also from state-approved textbooks. Topics covered in the test were *simple and compound interest*, *hire purchase*, *inflation* and *exchange rates*. The choice of topics was motivated by the perceived challenging nature of the topics for Grade 10 mathematics learners. When teachers were asked about topics that gave learners problems, they listed topics in financial mathematics. In addition, it was assumed that since the topics presented tasks that link mathematics to real-life issues they would stimulate and support the idea of a group approach largely embedded in mathematics lessons in experimental schools. The FMAT was developed through months of iterative processes of acquiring existing items from classroom teachers and state-approved textbooks, obtaining feedback from subject specialists and advisers, and conducting repeated content validity assessments. Local subject specialists, teachers and heads of mathematics departments from participating schools helped to revise test items to ensure they were aligned to learners' realistic context. The test consisted of five sections that addressed each of the selected

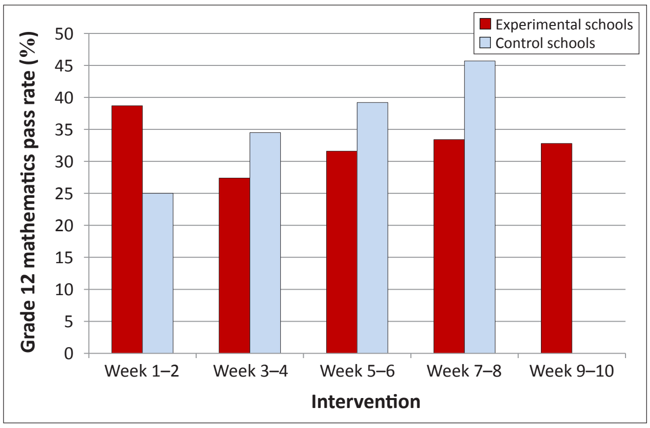

FIGURE 1: Intervention program for paired schools in the study.

topics. For example, an inflation-related item in one of the sections was given as follows:

4. Your brother wins a LOTTO competition and decides to invest R50 000 now. He secures an interest rate of 9% p.a. compounded annually. The inflation rate is currently running at 12% p.a.
   4.1 What will the future value of your brother's money be in 15 years from now?
   4.2 Due to inflation, what money will have the same buying power as R50 000 in 15 years' time?
   4.3 By how much will the buying power of your brother's money have declined after 15 years?

An example of a compound interest test item was:

2.3 Calculate the compound interest on a loan of R800 at 7% p.a. if the interest is compounded half yearly.

The FMAT had a total mark score of 60. The same test was written by all learners in both conditions before and after the investigation. The pre-test determined learners' initial performance status before intervention. A post-test was given at the end of a two-week intervention to examine the effects of a group approach (experimental group) and teachers' conventional teaching (control group) on the performance of learners.

In addition, the first author administered an instrument to investigate the influence of a group learning approach on the cognitive load of learners in experimental schools. The researchers adopted a self-reporting instrument developed by Paas, Van Merriënboer and Adams (1994). This instrument uses a post-test questionnaire in which test takers are asked to report the amount of mental effort (cognitive load) invested in performing problem-solving tasks in a test; there are nine choices. Each of the choices was presented in the learners' answer booklets or script immediately following each session of the FMAT (pre-test and post-test). The first measure of learners' cognitive load served as a baseline measure before intervention. The second measure served as an indication of the influence of a group approach on learners' cognitive load, and related problem-solving performance.

Finally, to gain access to learners' conceptions of whether or not working in groups, and also in non-group learning environments in control schools, enhanced their performance the first author did post-intervention semi-structured interviews. Two learners in each of the nine participating schools ($N = 18$) were purposively sampled for the interviews to provide their opinions on the style of teaching they followed in their respective schools. The following sampling criteria were established to select learners from both groups for interviews: (1) learners' performances in post-test (all performance categories were represented) and (2) quality of involvement during lessons (learners who were observed to participate minimally or maximally were equally selected). Interviews enabled us to elicit first-hand, in-depth primary data from learners. For the study of classroom practice and forms of classroom interactions that characterise approaches to FMAT-related worksheets, we asked all four teachers in control schools to allow the first author to observe their lessons only once. The task of the first author was to observe whether teachers' conventional teaching approaches followed a model of learner-centeredness or teacher-centredness. Using the descriptions presented earlier, a group approach was classified as a learner-centred approach, and teacher-dominated lesson was classified as a teacher-centred approach.

Assessment of measurement properties

The validity assessment for the achievement test (FMAT) was achieved through an expert panel in mathematics education and research that evaluated the suitability of the test for the purpose of the study. The panel provided valuable feedback as well as validity ratings for the test items, which were used to determine and improve content validity of the test for the study sample. For the reliability, Spearman-Brown's prophesy formula was used to determine the internal consistency of the FMAT. The formula yielded a reliability index of $r = 0.92$, indicating that the test was capable of producing similar results on repeated use. The first author conducted semi-structured interviews and lesson observations for consistency. A script with specific questions was used to standardise the interviews. Evidence of convergent validity was found when comparing interview results and observation data, both of which yielded a high correlation agreement of 0.87.

Instruction and data collection

In experimental schools the first author created group approach learning environments that dynamically altered the conventional roles of teachers and learners, and provided opportunities for collaborative discussions as suggested by Yackel and Cobb (1996). These authors talk about social norms that characterise effective collaborative or a group approach in classrooms. According to Yackel et al. (1991), social norms are general classroom expectations such as cooperation in problem solving and persistence on personally challenging problems. Social norms may 'regulate mathematical argumentation and influence the learning opportunities' (Yackel & Cobb, 1996, p. 461), particularly in a group learning environment. To form constructive participation structures and to emphasise Yackel and Cobb norms for effective group discussion, argumentation and reflection in experimental schools the first author arranged desks in groups of five to seven participants to facilitate learner interaction (see Dhlamini, 2012; Dhlamini & Mogari, 2011, 2012b). Each group consisted of learners who represented different levels of achievement. In each group, learners were encouraged to participate by explaining their problem-solving methods, and being willing to offer alternative problem-solving explanations. They were also encouraged to use their fellow group members' solution activities as prompts to develop group solutions. This meant that they would not just accept other learners' solutions as rigid, but could challenge, argue and critique solutions given to them by fellow group members. Learners in experimental schools were encouraged to be prepared and ready to share problem-solving knowledge with fellow group members, listen attentively when other group members presented their problem knowledge and be supportive of others. Also, in each group the first author appointed a leader, an explainer and a recorder. The teacher only provided explanations when they were required to do so.

In addition, in experimental schools the potential of more robust interaction was exploited with several worked-out problem examples in financial mathematics that were given to groups in worksheets (e.g. see Figure 2). Worked-out examples are a set of problem-related examples that presents an instructional step-by-step guideline on how to solve a problem (Dhlamini, 2011; Dhlamini & Mogari, 2011). Because worksheets were developed in collaboration with the four control schools teachers who participated in the study, both experimental schools and control schools were exposed to identical worksheet tasks. However, the mode of presenting the worksheet tasks varied between the two groups. In experimental schools, whilst group members studied steps in worked-out examples the first author walked from one group to another to ensure that learners understood the task assigned to them and adhered to the roles that the first auther assigned to them. Essentially, the first author also wanted to determine whether the group learning dynamics of listening, writing, answering, questioning and critically assessing contributions were taking place. According to Lai (2011), when teachers walk and circulate amongst groups, learners are more engaged and discussions are more fruitful.

An example of a worksheet task is presented in Figure 2. In prompting responses the first author asked questions such as: 'What helps you to choose the correct formula in Step 1?'; 'What is the difference between the notations 'A' and 'P' in our formulae?'; 'Can you summarise the steps in the worked-out example?' After working in groups and discussing the worked-out solutions learners received more tasks to solve, such as the following example: 'An amount of R1 200 accumulates to R2 600 after 3 years. Find the interest rate if the investment earned simple interest.'

When solving new tasks the first author encouraged learners to work independently in order to apply some of the ideas they discussed in groups. They were advised to seek assistance from group members only when they encountered challenges. Whilst they interacted in groups the first author asked questions such as: 'What do you think about what your fellow learner said?'; 'Do you agree with your fellow group member?'; 'Do you think you can learn from other learners?' Responses to these questions were considered to indicate learners' views about a group approach to mathematics. On the last day both the experimental group and control group wrote a post-test to measure the comparative effects of the instructional methods on learners' performance. The post-test had exactly the same questions as the pre-test.

Results and data analysis

Biographical data

At the beginning of the study all learners completed a demographic questionnaire. Of the 783 (322 boys and 461 girls), 724 (92.5%) supplied information on their ages. The mean age was computed only from returned forms. The ages of the learners ranged from 15 years to 19 years (M = 16.45; SD = 1.25). Table 1 shows that most participants (31.35%) were 16 years old.

Table 2 presents data that were collected to verify the suitability of schools to participate in the study. Schools are identified by the letter E (experimental school) or

TABLE 1: A comparison of the personal demographic details of participants.

Sample	Demographic details	N	Percentage
Learners	Boys	281	
	Gils	443	
Age groups	15 years	196	27
	16 years	227	31
	17 years	143	20
	18 years	96	13
	19 years	62	9
Total number of learners	-	**724**	-

N, number of learners.

Problem: How much money was invested five years ago if the value of the investment is currently R7000? The interest rate was 8% per annum simple interest.

Tools: **Formulae:** **Simple interest formula:** $A = P(1 + in)$

Compound interest formula: $A = P(1 + i)^n$

Notations:

P = Present value of the investment (original amount at the beginning);
A = Accumulated amount (future value) of the investment after n period;
n = Time period;
$i = \frac{r}{100}$ for the simple interest rate r%

Steps	Step-by-step explanation
$A = P(1 + in)$	**Step 1:** Choose the correct formula by using key words 'simple' and 'compound' in problem.
$A = 7000$; $P = ?$; $i = 0.08$; $n = 5$	**Step 2:** Arrange data by matching each value in problem to the correct symbol.
$7000 = P(1 + 0.08 \times 5)$	**Step 3:** Substitute data in the formula without changing the arrangement of the formula.
$7000 = P(1 + 0.4)$	**Step 4:** Work on more complicated side and apply BODMAS rule. Start by multiplication inside the bracket.
$7000 = P(1.4)$	**Step 5:** Add inside the bracket.
$\frac{7000}{1.4} = P$	**Step 6:** Divide both sides by 1.4 to make P the subject of the formula.
$P = 5000$	**Step 7:** Simplify and solve for P.

FIGURE 2: A sample of a worked-out problem-solving example.

TABLE 2: School and teacher profiles.

Group	School code	Teachers' years of teaching mathematics	Number of Grade 10 mathematics learners	Grade 12 pass rate for mathematics in the year preceding the study
Experimental group (*N* = 413)	E1	13	90	39%
	E2	21	74	27%
	E3	8	101	32%
	E4	18	67	33%
	E5	14	81	33%
Control group (*N* = 370)	C1	17	133	25%
	C2	23	71	35%
	C3	6	80	39%
	C4	13	86	46%

N, number of learners; E, experimental school; C, control school.

C (control school); hence E4 represents the fourth school in the experimental group.

Table 2 shows that all participating schools performed poorly in the Grade 12 examinations for mathematics in the year preceding the study. The performance of participating schools ranged between 25% (school C1) and 46% (school C4) (*M* = 34.3; *SD* = 6.3).

Achievement test (Financial Mathematics Achievement Test)

Quantitative results and analysis are provided to determine quantitative effects of the first author's group-related teaching approach and teachers' conventional lessons on learners' performance. In addition, the results of the quantitative analysis are used to answer the research question for the study. Of all learners (*N* = 783), 706 (90.2%) participated in all experimental activities, including attending all lessons during the two-week intervention, participating in classroom activities in both groups and writing the pre-test and post-test.

Based on the total score obtainable, learners were designated as low-performing (below 24), average-performing (between 24 and 42), and high-performing (above 42). The pre-test mean score of experimental schools was 20.9 whilst the mean score of the control schools was 22.0. The differential between these two means was 1.1 marks. The small differential suggests baseline equivalence of the two groups before intervention, thus confirming the earlier classification of learners as low-performers based on the Grade 12 results of their schools.

Post-test results suggested greater improvement in experimental schools, where a group approach was implemented. Here the mean score of the experimental schools was 33.3 (*SD* = 4.21; *N* = 378), whilst the mean score of control schools was 25.8 (*SD* = 4.10; *N* = 328). Because of the non-random sampling techniques employed in the study a one-way analysis of covariance (ANCOVA) was computed in order to test for statistical difference amongst all the variables and covariates. Pre-test scores were entered as the covariate and post-test scores as the dependent variable.

The phrase *between-subjects* (or within-learners) in Table 3 represents a measure of how much the FMAT scores of each learner tended to change (or vary) over the period of two weeks; this is measured by comparing the scores of the pre-test and post-test. The results in Table 3 show that when the effect of pre-test scores is removed, the effect of the group approach becomes significant, as confirmed by $F(1.703) = 558.7$, $p < 0.05$. This means that the group approach, as implemented in the experimental schools, is superior to the conventional teaching approaches implemented in the control schools in substantially improving learners' performance in certain topics in Grade 10 financial mathematics.

TABLE 3: The test of between-subjects effects.

Source	*SS*	*Df*	*M*	*F*	Significance
Pre-test	42.1	1	42.1	2.5	0.1
Groups	9643.6	1	9643.6	558.7	0.00
Error	12 134.8	703	17.3	-	-

$p < 0.05$

TABLE 4: The mean of learners' cognitive load during the pre-test and post-test.

Test	Mean	*SD*	Minimum	Maximum
Pre-test (*N* = 378)	6.6	1.6	3.0	9.0
Post-test (*N* = 378)	3.3	1.6	1.0	7.0

N, number of learners.

Cognitive load measurement

At the end of both the FMAT pre-test and the FMAT post-test learners in experimental schools (*N* = 378) were given a self-rating questionnaire to measure the cognitive load they invested in performing problem-solving tasks before and after the implementation of a group approach. The instrument consists of a nine-point scale: extremely easy (1), very easy (2), easy (3), quite easy (4), neither easy or difficult (5), quite difficult (6), difficult (7), very difficult (8) and extremely difficult (9). The mean values of learners' cognitive load before and after the implementation of the group approach is given in Table 4.

The mean cognitive load of learners in Table 4 shows that prior to the implementation of a group approach learners rated problem-solving tasks between 'quite difficult' and 'difficult'. However, Table 4 also shows that after the implementation of a group approach learners' cognitive load was reduced as learners generally rated problem-solving tasks in the post-test as being 'easy' (*M* = 3.3; *SD* = 1.6).

Semi-structured interviews

Of the 18 learners selected for the semi-structured interviews, 17 agreed to be interviewed. Some of the questions asked during interviews are: 'What is your view of teaching mathematics using a group approach?'; 'How do you value information from other group members?'; 'Do you think it is a good idea to group together high-performing and low-performing learners in mathematics?'; 'Do you think this approach of teaching can improve your performance in mathematics?' Learners from experimental schools preferred to be taught in a group approach mode, and clearly advocated a learner-centred approach. Typical responses of the learners in experimental schools are sampled below:

Learner 1: Eh … this method is good because in my class we are not talking [*implying that the teacher explained everything during the lesson*], but here we are talking to our friends for answers. It is good because I get it better from my fellow learners.

Learner 2: A group approach is better than our school method because we also talk as learners. I like it when we sit and discuss our views in maths. This is the best method to teach maths.

Learner 3: We must do these types of problems in my class. Our teacher must listen to us as students because these are our problems.

In particular, learners emphasised the importance of verbal interaction between learners when solving mathematics tasks. In fact, Learner 1 emphasised that she 'gets it better' when the explanation emerged from a fellow group member. When participants were asked about the difference between an explanation from the teacher and one from a fellow learner, typical responses were:

Learner 4: I think it is easier to challenge my friend than a teacher.

Learner 5: Sometimes it is easier when we use our language as students because we understand each other.

Learner 6: I think because we are all not perfect, if we discuss our mistakes as students we can improve.

Responses suggest that learners preferred a learner-centred approach (group approach) in which they engage in opportunities to share and discuss their mathematics ideas. A different picture emerged from learners in control schools. Learners from control schools revealed that they were never given an opportunity to discuss mathematics ideas amongst themselves. Most of the learners emphasised that in their schools they were subjected to a didactic teaching approach. The term *didactic teaching approach* is used to represent a popular view by learners from control schools that teaching in control schools was teacher-dominated. In another round of questions learners were asked to describe aspects of a group approach and traditional teaching approaches that they thought accounted for each instruction's beneficial effect. Responses from experimental schools pointed to worked-out examples as an effective learning tool.

Learner 7: I like this method of teaching because you have taught us with examples.

Learner 8: Eh … I liked this method. It is good and train students' minds. It is good because we were given examples that helped us to work alone. We will pass maths now.

Learner 9: I think a group method was good because it works with things that we know and good examples which make us to be familiar to the problems.

The responses provided by Learner 8 and Learner 9 emphasise the importance of retention and familiarity in learning. To enhance the positive benefits of a group approach learners sat in groups to discuss worked-out examples. Although identical tasks were tackled in control schools, the main mode of teaching used by teachers did not replicate the performance observed in experimental schools.

Lesson observations

The results of the observations of teachers' lessons corresponded with the learners' opinions that the teaching styles in control schools largely constrained opportunities for effective classroom interaction. Observations of teachers' lessons showed that lessons in most control schools were similar and embraced the following characteristics: the classrooms represented a traditional teaching and learning setting in which desks are arranged linearly; teachers solely explained problem-solving steps and provided solutions to learners; minimal learner participation and interaction took place during the lesson (lessons were less interactive); learners worked independently at their desks during mathematics tasks; in most cases, only one example was provided to facilitate mathematics learning. Using the definition of traditional teaching approaches that was given earlier, it is reasonable to use the results of our classroom observations to conclude that teachers' instructional modes in control schools conformed to a didactic teaching approach, suggesting that they employed a group approach to a very limited extent.

Discussion and conclusion

This study investigated the effect of implementing a group approach on the mathematics performance of low-performing mathematics learners. In experimental schools mathematics lessons took the form of a group learning approach, whilst in control schools teachers followed conventional non-group teaching approaches. The quantitative results revealed that a group approach was more effective ($p < 0.05$) when compared with teachers' conventional instruction. This finding suggests that a group approach has the potential to influence the academic achievement of learners in mathematics. The results of this study are not unique. Davidson (1985) reviewed about 80 studies in mathematics that compared learner performance in group settings versus whole-class conventional instruction. This author reported that in over 40% of the reviewed studies, learners in group learning approaches significantly outscored the control learners on individual mathematical performance measures.

More recently Kirschner et al. (2009) observed that learning by an individual is less effective and efficient than learning by a group of individuals during mathematics tasks. In terms of cognitive load theory, arranging learners in groups for learning purposes provides an opportunity to deal effectively with the limitations of working memory at individual level. This is important because within a group approach environment members have more processing capacity and so the construction of task-related schemas is promoted (Kirschner et al., 2009).

We acknowledge that the design of our investigation permitted the emergence of two independent variables within our study, namely a group learning approach and a worked-out examples approach. Hence, one of the challenges in interpreting the results of our investigation is separating the influence of these variables on the outcome of our experiment. To address this issue, and further provide a justification to moderate the confounding influence of the worked-out examples approach on learners' performance, we explored the following considerations: (1) a relatively larger component of the methodological setup of our investigation was hinged on the group approach rather than on the worked-out example approach and (2) there is a considerable empirical evidence to support the claim that a group learning approach promotes learner performance and long-term retention of the studied material, essentially in learning environments that put exclusive emphasis on this approach (see Lai, 2011; Whicker, Bol & Nunnery, 1997). Given these considerations, it may be reasonable to conclude that when learners in experimental schools discussed worksheet-related examples within a group approach context, they gained greater understanding of mathematics tasks, and were able to retain this knowledge and understanding longer.

Another confounding issue relates to the time span within which this investigation was conducted. It may be argued that a two-week period may not suffice to influence learners' problem-solving performance. The instructional design that was implemented in this study, which largely embraced aspects of cognitive load theory, yielded the outcome that successfully overcame the posing constraints of time. The results of this study provide substantial evidence to suggest that learners' problem-solving performance could be positively influenced over a short period of time (see Table 4). However, this study shows that this is possible when instructional design is aligned to learners' cognitive architecture to moderate the effect of extraneous problem-solving demands (cognitive load) on learners' performance (see also Dhlamini, 2012). Within group approach learning environments that were systematically constructed in experimental schools, learners' cognitive load was therefore reduced within a relatively short time span. Hence, this study shows that group approach instruction that is aligned to the principles of cognitive load theory may speedily accelerate learners' problem-solving performance.

We therefore emphasise that the outcome of our investigation was largely influenced by the type of instructional modes that were employed in both groups during the experiment. One of the learners commented: 'Working in a group also helped me to understand the example steps in worksheets better'. In our study worked-out examples were presented as mathematics tasks, not as a form of instruction. Worked-out examples worksheets served as springboards to group discussions. Therefore, the accelerated examples-related performance that is also reported by the learner in previous sentences could be linked to the fact that learners in experimental schools worked in groups. Despite these observations, we acknowledge that a further investigation with a modified methodological design should be conducted to explicitly demarcate the effects of a group learning approach and a worked-out examples approach on the mathematics performance of participants.

In conclusion, we acknowledge that the teaching of mathematics has often been viewed as an isolated, individualistic or competitive activity in which one sits and struggles alone to solve problems. Hence, at the beginning of this study we asked: What is the effect of using a group approach on the performance of learners when teaching certain topics in Grade 10 financial mathematics? The results of this study, and those of other related studies, provide some hope that group learning may still have a place in mathematics teaching. Specifically, the results of this study provide some evidence to suggest that a group approach may be an effective way to teach certain topics in Grade 10 mathematics.

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Competing interests

We declare that we have no financial or personal interest(s) that may have inappropriately influenced us when writing the article

Authors' contributions

J.D. (University of South Africa) carried out the research that generated the data used in this article and drafted the manuscript. D.M. (University of South Africa) provided guidance and oversight during the study and finalised the manuscript for publication.

References

Barnes, M.S. (2003). *Collaborative learning in senior mathematics classrooms: Issues of gender and power in student-student interactions*. Unpublished doctoral dissertation. University of Melbourne, Melbourne, Australia.

Beers, P.J., Boshuizen, H.P.A., & Kirschner, P.A. (2007). The analysis of negotiation of common ground in CSCL. *Learning and Instruction, 17*, 427–435. http://dx.doi.org/10.1016/j.learninstruc.2007.04.002

Chen, M. (2003). Cognitive load theory and its implications for instruction. *Educational Review of National Kaohsiung Normal University, 21*, 29–51.

Davidson, N. (1985). Small group learning and teaching in mathematics: A selective review of the research. In R.E. Slavin, S. Sharan, S. Kagan, R. Hertz-Lazarowitz, C. Webb, & R. Schmuck (Eds.), *Learning to cooperate, cooperating to learn* (pp. 211–230). New York, NY: Plenum Press.

Davidson, N., & Kroll, D.L. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education, 22*, 362–365. http://dx.doi.org/10.2307/749185

Dhlamini, J.J. (2011). Context-based problem solving instruction to induce high school learners' problem solving skills. In H. Venkat, & A. Essien (Eds.), *Proceedings of the 17th Annual National Congress of the Association for Mathematics Education of South Africa*, Vol. 1 (pp. 135–142). Johannesburg: AMESA. Available from http://www.amesa.org.za/AMESA2011/Volume1.pdf

Dhlamini, J.J. (2012). *Investigating the effect of context-based problem solving instruction on the performance of learners.* Unpublished doctoral dissertation. University of South Africa, Pretoria, South Africa. Available from http://hdl.handle.net/10500/9739

Dhlamini, J.J., & Mogari, D. (2011). Using a cognitive load theory to account for the enhancement of high school learners' mathematical problem solving skills. In D. Mogari, A. Mji, & U.I. Ogbonnaya (Eds.), *Proceedings of the ISTE 2011 International Conference: Towards Effective Teaching and Meaningful Learning in MST* (pp. 309–326). Kruger National Park, Mpumalanga: University of South Africa.

Dhlamini, J.J., & Mogari, D. (2012a). Designing instruction to promote mathematical problem solving performance of high school learners. In D. Mogari, A. Mji, & U.I. Ogbonnaya (Eds.), *Proceedings of the ISTE International Conference on Mathematics, Science and Technology Education: Towards Effective Teaching and Meaningful Learning in MST* (pp. 444–449). Kruger National Park, Mpumalanga: University of South Africa.

Dhlamini, J.J., & Mogari, D. (2012b). Using a context-based problem solving instruction to promote social justice in high school mathematics classrooms. In S. Nieuwoudt, D. Laubscher, & H. Dreyer (Eds.), *Proceedings of the 18th Annual National Congress of the Association for Mathematics Education of South Africa,* Vol. 1 (pp. 108–119). Potchefstroom: AMESA. Available from http://www.amesa.org.za/AMESA2012/Volume1.pdf

Edwards, J., & Jones, K. (2001). Exploratory talk within collaborative small groups in mathematics. In J. Winter (Ed.), *Proceedings of the British Society for Research into Learning Mathematics,* Vol. 21(3) (pp. 19–24). Available from http://www.bsrlm.org.uk/IPs/ip21-3/BSRLM-IP-21-3-4.pdf

Engle, R.W. (2010). Role of working memory capacity in cognitive control. *Current Antropology, 51*(1), S17–S26. http://dx.doi.org/10.1086/650572

Erichsen, E.A., Goldenstein, C., & Kaiser, L.M.R. (2011). Fostering collaborative and interdisciplinary research in adult education: Interactive resource guides and tools. *Sage Open*, *1*(1). http://dx.doi.org/10.1177/2158244011403804

Gaigher, E. (2006). *The effect of a structured problem solving strategy on performance and conceptual understanding of physics: A study in disadvantaged South African schools*. Unpublished doctoral dissertation. University of Pretoria, Pretoria, South Africa. Available from http://upetd.up.ac.za/thesis/available/etd-02022006-160908/

Gay, L.R., Mills, E., & Airasian, P. (2011). *Educational research: Competencies for analysis and application*. (10th edn.). Upper Saddle River, NJ: Pearson Education.

Johnes, G. (2006). Didacticism and educational outcomes. *Educational Research and Reviews, 1*(2), 2328. Available from http://www.academicjournals.org/err/PDF/Pdf2006/May/Geraint.pdf

Johnson, D.W., Johnson, R.T., & Stanne M.B. (2001). Impact of goal and resource interdependence on problem solving success. *The Journal of Social Psychology, 129*, 621–629. http://dx.doi.org/10.1080/00224545.1989.9713780

Kirschner, F.C. (2009). *United brains for complex learning: A cognitive load approach to collaborative learning efficiency*. Unpublished doctoral dissertation. Inter-university Centre for Educational Research, Open University, Maastricht, The Netherlands. Available from http://dspace.ou.nl/handle/1820/2391

Kirschner, F.C., Paas, F., & Kirschner, P.A. (2009). A cognitive load approach to collaborative learning: United brains for complex tasks. *Educational Psychology Review, 21*, 31–42. http://dx.doi.org/10.1007/s10648-008-9095-2

Kirschner, F.C., Paas, F., Kirschner, P.A., & Janssen, J. (2011). Differential effects of problem solving demands on individual and collaborative learning outcomes. *Learning and Instruction, 21*, 587–599. http://dx.doi.org/10.1016/j.learninstruc.2011.01.001

Kirschner, P.A. (2002). Cognitive load theory: Implications of cognitive load theory on the design of learning. *Learning and Instruction, 12*, 1–10. http://dx.doi.org/10.1016/S0959-4752(01)00014-7

Kirschner, P.A., Beers. P.J., Boshuizen, H.P.A., & Gijselaers, W.H. (2008). Coercing shared knowledge in collaborative learning environments. *Computers in Human Behaviour, 24*, 403–420. http://dx.doi.org/10.1016/j.chb.2007.01.028

Lai, E.R. (2011). *Collaboration: A literature review. Research report*. New York, NY: Pearson. Available from http://www.pearsonassessments.com/hai/images/tmrs/Collaboration-Review.pdf

Munneke, L., Andriessen, J., Kanselaar, G., & Kirschner, P.A. (2007). Supporting interactive argumentation: Influence of representational tools on discussing a wicked problem. *Computers in Human Behaviour, 23*(3), 1072–1088. http://dx.doi.org/10.1016/j.chb.2006.10.003

Paas, F.G.W.C., Van Gog, T., & Sweller, J. (2010). Cognitive load theory: New conceptualizations, specifications, and integrated research perspectives. *Educational Psychology Review, 22*(1), 115–121. http://dx.doi.org/10.1007/s10648-010-9133-8

Paas, F.G.W.C., Van Merriënboer, J.J.G., & Adam, J.J. (1994). Measurement of cognitive load in instructional research. *Perceptual and Motor Skills, 79,* 419–430. http://dx.doi.org/10.2466/pms.1994.79.1.419, PMid:7808878

Van Boxtel, C.A.M., Van der Linden, J., & Kanselaar, G. (2000). Collaborative learning tasks and the elaboration of conceptual knowledge. *Learning and Instruction, 10*, 311–330. http://dx.doi.org/10.1016/S0959-4752(00)00002-5

Whicker, K.M., Bol, L., & Nunnery, J.A. (1997). Cooperative learning in the secondary mathematics classroom. *The Journal of Educational Research, 91*(1), 42–48. http://dx.doi.org/10.1080/00220679709597519

Yackel, E., & Cobb, P. (1996) Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477. http://dx.doi.org/10.2307/749877

Yackel, E., Cobb, P., & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education, 22*(5), 390–408. http://dx.doi.org/10.2307/749187