Students' understanding of geometry terminology through the lens of Van Hiele theory

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After a long six-year lapse, the Curriculum and Assessment Policy Statement introduced in 2012 included geometry as part of the South African Grade 12 Mathematics Paper 2. The first cohort of matriculation students wrote Paper 2 in 2014. This article reports on the understanding of geometry terminology with which a group of 154 first-year mathematics education students entered a rural South African university in 2015; 126 volunteered to be part of the study. Responses to a 60-item multiple-choice questionnaire (30 verbally presented and 30 visually presented items) in geometry terminology provided the data for the study. A concept's verbal description should be associated with its correct visual image. Van Hiele theory provided the lens for the study. An overall percentage mean score of 64% obtained in the test indicated that the majority of the students had a fairly good knowledge of basic geometry terminology. The students obtained a percentage mean score of 68% on visually presented items against that of 59% on verbally presented items implying a lower level thinking as per Van Hiele theory. The findings of this study imply a combination approach using visual and verbal representations to enhance conceptual understanding in geometry. This has to be complemented and supplemented through scaffolding to fill student teachers' content gap.

Introduction

Internationally, identifying the challenges in the preparation of mathematics teachers is a growing field of research as it is one of the most urgent problems faced by those who wish to improve student learning. For example, the cross-national study on the preparation of middle school mathematics teachers by Schmidt (2013) and the review of 26 studies on prospective teachers' content knowledge in geometry and measurement by Browning, Edson, Kimani and Aslan-Tutak (2014) are some of the studies that show that the international mathematics education community is trying to address some of the issues in pre-service teacher education. Teachers are the determining factors of successful educational change within an education and training system so that advancement of teachers through education is a method to access optimal and successful educational changes (Mostafa, Javad, & Reza, 2017). The South African education system is confronted with the under-preparedness of teachers particularly in the teaching of mathematics in rural areas (Aldridge, Fraser, & Ntuli, 2009). This can be attributed to the ongoing curriculum changes in South Africa since its democratic inception in 1994. The education community has seen many policy revisions, modifications and reformations such as Curriculum 2005 in 1998, the National Curriculum Statement in 2001 and the Revised National Curriculum Statement in 2002 (King, 2003). The Revised National Curriculum Statement came in to effect in the Further Education and Training Band (FET) in 2006, where Euclidean geometry was excluded from the compulsory mathematics curriculum component (see Alex & Mammen, 2014). Later on, a further revision of the curriculum called Curriculum and Assessment Policy Statement came into effect in the FET phase in Grade 10 in 2012 and the first cohort of matriculation students wrote the Mathematics Paper 2 with Euclidean geometry as a compulsory topic in 2014. The year 2013 was the very last year when the students wrote geometry as part of the optional Paper 3 for Mathematics in Grade 12. For that reason, in this study, the year of passing Grade 12 was taken as a factor contributing to the performance in geometry. The purpose of this study was to investigate the understanding of geometry terminology with which pre-service student teachers entered a rural university in 2015. Insight of this nature is important for effective remedial teaching measures.

Conceptual understanding in school mathematics and geometry

In South Africa, one of the aims of teaching mathematics is to develop an understanding of spatial concepts and relationships (Department of Education, 2003). The idea of re-conceptualising the approach to geometry teaching and learning was placed in the foreground of the introduction of

Curriculum 2005 in 1998 by the South African National Ministry (King, 2003). One of the aims as stipulated in the National Curriculum Statement Grades R–12 is that teachers of mathematics need to produce learners who are able to communicate effectively using visual, symbolic or language skills in various modes (Department of Basic Education, 2011). Learning with understanding has been problematic, especially in the domain of mathematics (Stylianides & Stylianides, 2007). Learning with understanding has increasingly received attention from educators and psychologists, and has progressively been elevated to one of the most important goals for all students. In order to prepare mathematically literate citizens for the 21st century, classrooms need to be restructured so that mathematics can be learned with understanding (Carpenter & Lehrer, 1999). French (2004) emphasises that students' general mathematical competencies have been closely linked to their geometric understanding. According to Couto and Vale (2014), the development of geometrical thought is an important auxiliary to solving problems in students' daily lives. It is important to develop the skills to see, analyse and think about the spatial objects and their images within the child (Battista, 2007). This shows the importance of geometry in the overall mastery of mathematics, and further explains why geometry assumes a dominant place in the school mathematics curricula of many countries (Atebe, 2008). In order to meaningfully teach mathematics in general and geometry in particular, developing each student's conceptual understanding is important.

Concept is an element of understanding and knowledge (Öksüz, 2010). Mathematics education should include appropriate emphasis on the teaching of conceptual understanding of mathematics. According to Kilpatrick, Swafford and Findell (2001), a significant indicator of conceptual understanding is the ability to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. Suh (2007) stressed the use of representations to foster conceptual understanding. According to Cunningham and Roberts (2010), when in the process of trying to recall a concept, it is not usually the concept definition that comes to a student's mind but the prior experiences with diagrams, attributes and examples associated with the concept. Conceptual knowledge of geometrical concepts goes further than the skills required to manipulate geometric shapes (Luneta, 2015). It is crucial that future teachers know the basic concepts well in order to understand complex concepts (Couto & Vale, 2014). According to Brown, Cooney and Jones as cited in Cunningham and Roberts (2010), pre-service elementary teachers did not possess the level of mathematical understanding that was necessary to teach at the level recommended by the National Council of Teachers of Mathematics (NCTM). The situation is similar in South Africa. For example, while Bennie (1998) reported lack of conceptual understanding among teachers, Atebe and Schäfer (2010) reported on the lack of conceptual understanding in mathematics in general and geometry in particular among school learners. As such, there was a need for additional research which warranted this study. Identifying students' prior knowledge, before commencing teaching had been considered as a good measure to pitch the learning and teaching at an appropriate level to enhance learning.

Teaching and learning of geometry terminology

Understanding geometry is an important mathematical skill since the world in which we live is 'inherently geometric' (Clements & Battista, 1992, p. 420). Geometry focuses on the development and application of spatial concepts through which children learn to represent and make sense of the world (Thompson, 2003). Geometry is an essential part of the South African mathematics curriculum (Alex & Mammen, 2016). According to Genz (2006), evidence from a variety of sources makes it clear that students at junior school level are not learning geometry concepts appropriately in order to prepare them for success in their high school geometry course. 'The most basic type of knowledge in any particular field is its terminology' (Bloom, 1956, p. 63). De Villiers, as cited by Feza and Webb (2005), suggests that acquisition of technical terminology is the key to success in learning geometry. Students need to acquire the correct technical terms and be able to use them correctly to communicate their ideas about concepts in geometry (Atebe & Schäfer, 2010, p. 54). Sherard (1981) states that our basic speaking and writing vocabularies are rich in many geometric terms, such as point, line, angle, parallel, perpendicular, plane, circle, square, triangle, and rectangle and this geometric terminology helps us to communicate our ideas to others in precise forms. Geometry is slotted as an important school subject because it provides perspectives for developing students' deductive reasoning abilities and the acquisition of spatial awareness (NCTM, 1989). Jones (2002) suggests that geometry helps the students to develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. In the literature, spatial sense, spatial perception, spatial insight, spatial visualisation and spatial orientation have been used for reference to spatial skills in geometric thinking (Bennie, 1998). According to Clements and Battista (1992, p. 444), 'spatial ability is important in students' construction and use of non-geometrical' concepts. Spatial ability plays a very important part in the development of geometric concepts and their representations (Nickson, 2000). The NCTM draft Standards 2000 document suggests that mathematics instruction programmes should pay attention to geometry and spatial sense so that students, among other things, use visualisation and spatial reasoning to solve problems both within and outside of mathematics (Mathematics Learning and Teaching Initiative [MALATI], 1997). Correct terminology of concepts is necessary to avoid misconceptions and confusion. The pre-service training period of teachers is ideal to ensure grounding in correct geometric terminology. Couto and Vale (2014) state that in Portugal, the mathematics programme for basic teacher education stresses several factors such as: the visualisation and comprehension of properties of geometrical figures, the understanding of how important these are for the development of the student's spatial awareness and the introduction of the study of geometrical transformations from early years, and how it progressively widens. According to Gal and Linchevski (2010), in the professional development of teachers, one should examine ways to incorporate theories of visual perception as well as their use in analysing difficulties. Furthermore, as Ndlovu, Wessels and De Villiers (2013) point out, the quality of teachers determines the quality of an education system.

Van Hiele theory on learners' understanding of geometry

How children develop their understanding of geometry and spatial sense has been an area of research over the past 60 years (Alex, 2012). During 1997, MALATI tried to re-conceptualise the teaching and learning of geometry in South African schools (Bennie, 1998). For that re-conceptualisation to happen and to propose changes to the curriculum, MALATI felt that a model to understand the geometric thinking of learners would be needed (King, 2003). The group found that the Van Hiele model of geometric thinking would be a framework to understand the geometric thinking of learners. The theory of geometrical thinking proposed by Van Hiele in the 1950s suggested five sequential and discrete levels of thought a learner passes through, namely Visualisation (recognising and naming figures), Analysis (describing the attributes of shapes), Informal Deduction (classifying and generalising by attributes), Deduction (developing proofs using axioms and definitions) and Rigor (working in various geometrical systems) (Alex, 2012). The Van Hiele levels 'explain the understanding of spatial ideas and how one thinks about them' (Luneta, 2015, p. 11). The levels are hierarchical and each level is characterised by its own language and vocabulary (Van Hiele, 1986). The first two levels (Visualisation and Analysis) are particularly important for the discussion in this article. In the former, the learners reason about basic geometric concepts such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components (Burger & Shaughnessy, 1986). In the latter, learners begin to identify properties of shapes and learn to use appropriate vocabulary related to properties (Teppo, 1991). Van Hiele theory has much influence on the argument of Cunningham and Roberts (2010) that in the process of trying to recall a concept, prior experiences with diagrams, attributes and examples associated with the concept come to a student's mind before the concept's definition. A learner operating at Analysis level will be able to master the visual and verbal attributes better than a learner operating at Visualisation level. According to Van Hiele (1986), teaching of geometry is central to the development of logical thinking, a key element of mathematical understanding. Van Hiele asserted that teachers have a crucial role in the process of teaching and learning (Couto & Vale, 2014). The more a teacher knows about the way students learn, the more effective that individual will be in nurturing mathematical understanding (Swafford, Jones, & Thornton, 1997). The Van Hiele theory aims to explain how children learn geometrical concepts (Lim, 2011). According to Shulman (1987), teaching necessarily begins with a teacher's pedagogical content knowledge. According to Rossouw and Smith (1997), the rich base developed from research on Van Hiele levels and how students learn geometry is an important source of understanding teachers' pedagogical content knowledge of geometry teaching. The Van Hiele theory with this significant pedagogical implication in geometry thinking levels forms the basis of this study.

Research questions

The research questions investigated in this study are: (1) What was the conceptual understanding of basic geometrical terminology of the sample of 2015 entry level mathematics education students in the rural university in relation to the curriculum followed by the pre-2014 matriculants and matriculants of 2014? (2) Was there a statistical difference in the overall performance of students in matching the verbal description with their correct visual images? And (3) was there a statistical difference in the performance of the students with respect to the different concepts tested?

Methodology

This research adopted a positivist paradigm and a quantitative approach. This case study design mainly focused on matching the verbal description with their correct visual images in the geometrical concepts and terminology and as indicated earlier was a retrospective study.

Instrument

Euclidean geometry in South African schools is usually the figures in the plane (Atebe, 2008). The instrument used was a 60-item multiple choice questionnaire with four options from which the students were expected to choose the best answer. The questionnaire was constructed by Atebe (2008) and the researchers adopted it with permission. The aim was to explore students' understanding of some key technical terms frequently encountered in the teaching and learning of school geometry (Atebe, 2008). Atebe used the split-half method to check the reliability of the test construction and the Spearman-Brown reliability coefficient (r) calculated for the test was 0.87; thus, the test was found to be reliable as the sample of Atebe and the sample for this study were similar in nature. The items mainly tested the terminology associated with concepts about lines, angles, triangles, quadrilaterals and circles. There were two conceptually identical but structurally different sets of items for each of the 30 selected terminologies, one in a verbal form (with no diagrams), and the other in a visual form (with diagrams). In other words, the multiple choice questionnaire consisted of 30 verbally presented and 30 visually presented items. All the items in the questionnaire were then juggled around so that items in the homologous (i.e. identical) pair of items were separated far away from the other. The purpose of the identical pair was to determine whether a student who chose the correct verbal description of a geometric concept also chose the correct visual representation and vice versa.

For example, in Figure 1, Item 1 (verbal) and Item 10 (visual) form a homologous pair used in the questionnaire.

The instrument had two sections, the first one to gather biographic data and the second one consisted of the 60-item multiple choice questionnaire. The lead researcher administered the instrument during the first two-hour mathematics education lecture.

Analysis and results

General information of the students who participated in the study

The mean average age of the students who participated in the study was 22 years. Out of the 126 students who participated in the study, who were enrolled for the year 2015 in year 1, 88 (70%) passed matric before 2014 (pre-2014 matriculants) and 38 (30%) were matriculants of 2014.

Ethical considerations

This article reports on a study that was conducted in a historically disadvantaged rural university in the Eastern Cape. To conduct the study, permission was sought from the Head of the Department of Mathematics, Natural and Consumer Sciences Education. A permission letter was obtained and approved by the Faculty of Educational Sciences Ethics Committee. Entry level mathematics education students were informed of the purpose of the test and a request for voluntary participation was made. Out of the 154 students who enrolled for the course for the year 2015, 126 (86 male students and 40 female students) voluntarily took part in the study. It was agreed that anonymity and confidentiality of the data would be guaranteed. This information was also printed on the general information of the instrument and there was space for participants to sign for informed consent. There was no reward for participation.

Performance of entry level mathematics education students in the geometry terminology test

The scoring of the terminology section was calculated using Microsoft Excel 2013. Each item was awarded 1 mark and the total was 60 marks. For each student, the total was then converted to a percentage. The general performance of the students was calculated in terms of the overall percentage mean score as shown in Figure 2.

For research question (1), an overall percentage mean score of 64% obtained in the test indicated that the majority of the students (64%) in this study had a fairly good knowledge of basic geometric terminology. The study further aimed to determine the students' ability in visually presented and verbally presented terminology items. The students obtained a percentage mean score of 68% on visually presented items against a percentage mean score of 59% on verbally presented terminology items. This meant that the students' performance was better in dealing with visually presented terminology items than the verbally presented items for the same concept.

To answer research question (2), a further analysis was also done to find out the performance of students in the multiple choice questionnaire in relation to pre-2014 matriculants and the matriculants of 2014. Table 1 depicts the results.

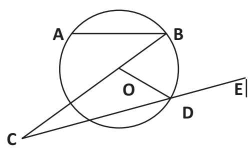
From Table 1 it can be noted that $p < 0.05$ and that the obtained t-value 6.748214 is significant, as $|t| > 1.96$ at $\alpha = 0.05$ for a two-tailed independent samples test. This shows that there is a significant statistical difference in the overall performance of the students in favour of matriculants of 2014.

From Table 2 it can be noted that $p < 0.05$ and that the obtained t-value 6.260318 is significant, as $|t| > 1.96$ at $\alpha = 0.05$ for a

1. What is the straight line that joins any two points on the circumference of a circle called?

   (A) an arc (B) a diameter (C) a radius (D) a chord

10. In the diagram, O is the centre of the circle. Which line segment represents a chord?



   (A) OD (B) AB (C) CE (D) CD

Source: Atebe, H.U. (2008). *Students' Van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa.* Unpublished doctoral dissertation, Rhodes University, Grahamstown, South Africa (p. 92). Retrieved from http://hdl.handle.net/10962/d1003662

FIGURE 1: A homologous pair.

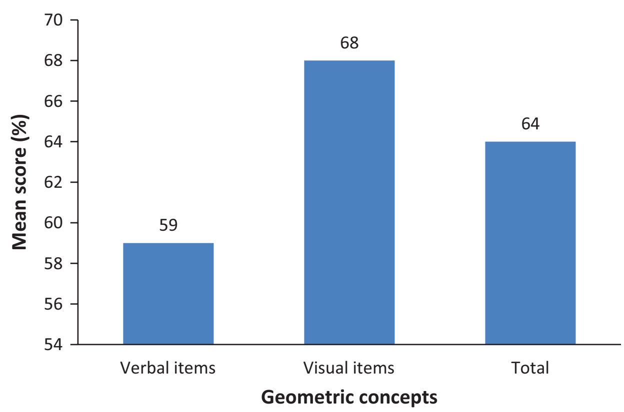


FIGURE 2: Performance of entry level mathematics education students in the geometry terminology test.

TABLE 1: Performance of students in the multiple choice questionnaire in relation to pre-2014 matriculants and the matriculants of 2014.

Year of passing matric	*n*	Mixed items (all) (mean score %)	SD
Before 2014	88	59	15
In 2014	38	76	12

Note: *df*, 126; *t*-value, 6.748214; *p*-value, 0.0000000053.

TABLE 2: Performance of students in the visually presented items in relation to pre-2014 matriculants of and the matriculants of 2014.

Year of passing matric	*n*	Visually presented items (mean score %)	SD
Before 2014	88	64	15
In 2014	38	79	11

Note: *df*, 126; *t*-value, 6.260318; *p*-value, 0.0000000079.

two-tailed independent samples test. This shows that there is a significant statistical difference in the performance in visually presented items in favour of matriculants of 2014.

From Table 3 it can be noted that $p < 0.05$ and that the obtained t-value 5.932766 is significant, as $|t| > 1.96$ at $\alpha = 0.5$ for a two-tailed independent samples test. This shows that there is a significant statistical difference in the verbally presented items in favour of matriculants of 2014.

From Table 4 it can be noted that $p < 0.05$ and that the obtained t-value 4.137707405 is significant, as $|t| > 1.96$ at $\alpha = 0.5$ for a one-tailed independent samples test. This shows that there is a significant statistical difference in the performance of the students in favour of visually presented items.

From Table 5 it can be noted that $p < 0.05$ and that the obtained t-value 2.319806649 is significant, as $|t| > 1.96$ at $\alpha = 0.5$ for a one-tailed independent samples test. This shows that there is a significant statistical difference in the performance of the students in favour of visually presented items.

Students' knowledge of different concepts in geometry

To address research question (3), students' mean scores in the terminology test were calculated separately for items on geometric terminology associated with the concepts in three categories: lines, circles, and triangles and quadrilaterals. Table 6 shows how the different concepts were asked in the terminology test.

The results were analysed using Microsoft Excel 2013 and are shown in Figure 3.

It was found that the students performed better in the terminology associated with lines (70%) followed by circles (64%) and the terminology in triangles and quadrilaterals were the worst performed (52%).

It was found that the matriculants of 2014 performed better in the terminology associated with lines (83%) followed by circles (75%) and the terminology of triangles and quadrilaterals (63%) than the pre-2014 matriculants who scored 64%, 60% and 48% respectively.

From Table 7 it can be noted that $p < 0.05$ and that the obtained t-value 6.131306938 is significant, as $|t| > 1.96$

TABLE 3: Performance of students in the verbally presented items in relation to pre-2014 matriculants of and the matriculants of 2014.

Year of passing matric	n	Verbally presented items (mean score %)	SD
Before 2014	88	54	17
In 2014	38	72	15

Note: df, 126; t-value, 5.932766; p-value, 0.000000090.

TABLE 6: The different concepts tested in the terminology test.

Concept	Question numbers	Total number of questions
Lines	4, 8, 13, 16–19, 21, 22, 25, 27, 31, 32, 34–37, 39–43, 49, 50, 52, 55, 58, 60	28
Circles	1–3, 5–7, 10, 14, 15, 23, 24, 29, 30, 38, 56, 59	16
Triangles and quadrilaterals	9, 11, 12, 20, 26, 28, 33, 44–48, 51, 53, 54, 57	16

Source: Adapted from Atebe, H.U. (2008). *Students' Van Hiele levels of geometric thought and conception in plane geometry: A collective case study of Nigeria and South Africa.* Unpublished doctoral dissertation, Rhodes University, Grahamstown, South Africa. Retrieved from http://hdl.handle.net/10962/d1003662

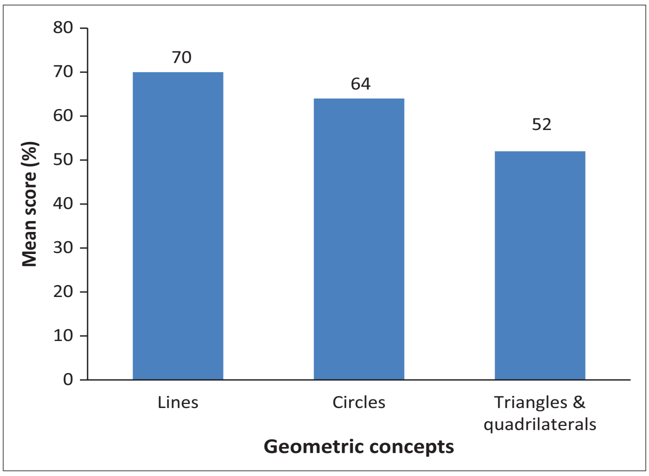


FIGURE 3: Performance of entry level mathematics education students in the multiple choice questionnaire according to concepts.

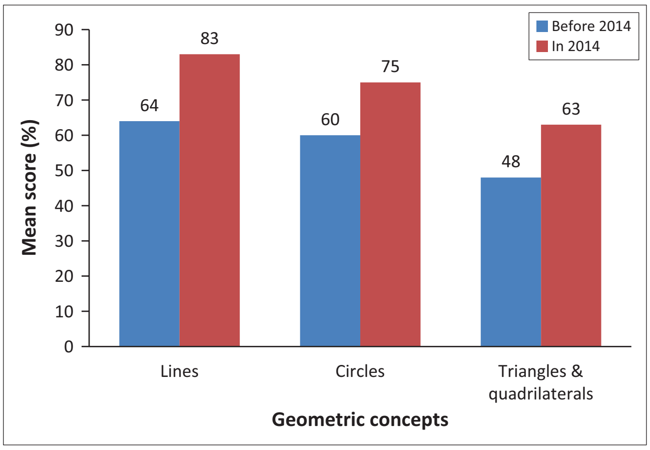


FIGURE 4: Performance of entry level mathematics education students in the multiple choice questionnaire according to concepts.

TABLE 4: Performance of pre-2014 matriculants in relation to the verbally presented and visually presented items in the test.

Year of passing matric	n	Verbally presented items (mean score %)	SD	Visually presented items (mean score %)	SD
Before 2014	88	54	17	64	15

Note: df, 88; t-value, 4.137707405; p-value, 0.0000450312.

TABLE 5: Performance of matriculants of 2014 in relation to the verbally presented and visually presented items in the test.

Year of passing matric	n	Verbally presented items (mean score %)	SD	Visually presented items (mean score %)	SD
In 2014	38	72	15	79	11

Note: df, 38; t-value, 2.319806649; p-value, 0.0077571090.

TABLE 7: Performance of students in the line concepts in relation to pre-2014 matriculants and matriculants of 2014.

Year of passing matric	*n*	Lines (mean score %)	SD
Before 2014	88	64	18
In 2014	38	83	15

Note: *df*, 126; *t*-value, 6.131306938; *p*-value, 0.0000000217.

TABLE 8: Performance of students in the circles concepts in relation to pre-2014 matriculants and matriculants of 2014.

Year of passing matric	*n*	Circles (mean score %)	SD
Before 2014	88	60	17
In 2014	38	75	15

Note: *df*, 126; *t*-value, 4.943971704; *p*-value, 0.0000071621.

TABLE 9: Performance of students in the triangles and quadrilaterals concepts in relation to pre-2014 matriculants and matriculants of 2014.

Year of passing matric	*n*	Triangles and quadrilaterals (mean score %)	SD
Before 2014	88	48	17
In 2014	38	63	16

Note: *df*, 126; *t*-value, 4.738459571; *p*-value, 0.0000067096.

at $\alpha = 0.5$ for a two-tailed independent samples test. This shows that there is a significant statistical difference in the performance of the students in the concept of lines in favour of matriculants of 2014.

From Table 8 it can be noted that $p < 0.05$ and that the obtained *t*-value 4.943971704 is significant, as $|t| > 1.96$ at $\alpha = 0.5$ for a two-tailed independent samples test. This shows that there is a significant statistical difference in the performance of the students in the circles concepts in favour of matriculants of 2014.

From Table 9 it can be noted that $p < 0.05$ and that the obtained *t*-value 4.738459571 is significant, as $|t| > 1.96$ at $\alpha = 0.5$ for a two-tailed independent samples test. This shows that there is a significant statistical difference in the performance of the students in triangles and quadrilateral concepts in favour of matriculants of 2014.

It is noted from Tables 1–5 and Tables 7–9 that the performance of the students who wrote geometry as part of their matriculation examination in the year 2014 outperformed the students who passed earlier than 2014 as the percentages for verbally presented items, visually presented items and all the concepts were higher than the students who passed matric before 2014.

Discussion

This study found that students' performance was better in dealing with visually presented terminology items than verbally presented ones. This could lead to the conclusion that the students in the study, although high school graduates, could probably be operating at lower Van Hiele levels of geometric thinking. According to Couto and Vale (2014), the development of geometrical thought is an important ancillary to solving problems in students' daily lives and the attainment of these ideas depends greatly upon the teacher and their knowledge. The performance of the student teachers of this study raises a concern in this respect and they are to be trained at university entry level in order to reach the expected levels of the Van Hiele theory.

In support of the Van Hiele theory, Couto and Vale (2014) state that geometrical thought is gradually developed in students starting with recognition of figures, moving on to differentiation up to the emergence of deductive reasoning. An earlier study by Alex and Mammen (2014) on the Van Hiele theory of geometrical thinking found that the majority of South African Grade 10 learners were at the Visualisation level. One of the characteristics of Visualisation level thinkers is that they reason about basic geometric concepts such as simple shapes, primarily by means of visual consideration of the concept as a whole without explicit regard to properties of its components. Other South African studies such as those of De Villiers and Njisane (1987), Siyepu (2005), Atebe (2008) and Luneta (2015) indicated that high school learners in general, and more especially Grade 12 learners, were functioning below the levels that were expected of them, that is, they were at the concrete (Visualisation) level rather than at the abstract level in geometry.

The results from this study were inconsistent with the study by Bozkurt and Koç (2012) on first-year elementary mathematics teacher education students where the majority of the participants (32%) could not provide a definition of a prism (word skills). Also, the results reported here were inconsistent with those of Couto and Vale (2014) on pre-service teachers in Portugal. Their study showed a weak performance in the test on issues addressing elementary knowledge of geometry where there were only 34% correct answers in knowledge and understanding of concepts and mathematical knowledge. The overall percentage mean score of 64% in the present study was inconsistent with Atebe (2008) in that the 64% was better than the mean score (47.85%) of the South African subsample of learners in his study. The results from the present study were also inconsistent with the study by Cunningham and Roberts (2010) where 23 elementary pre-service teachers were assessed on their ability to answer questions involving geometry concepts and a weak or limited understanding of certain concepts was reported. The results, however, were confirmation of the analyses by Luneta (2015) of 1000 Grade 12 scripts from 2012 in South Africa which followed the National Curriculum Statement, where most of the students made conceptual errors in questions in geometry.

The data show that the matriculants of 2014 outperformed, with a significant statistical difference, the pre–2014 matriculants. The present study revealed a very significant gap in the performance of the pre-service student teachers in geometry at the university entry level in favour of students who came through a compulsory geometry curriculum. This might also be due to the constraints in the secondary school mathematics curriculum originating from curriculum reforms. This is in support of the inference put forward by

Wilburne and Long (2010) that many pre-service teachers find that they never get the opportunity to really study the mathematics curriculum in depth but are expected to know and teach it with meaning in their student teaching and beginning of teaching experiences.

Conclusions

This study investigated the knowledge of basic geometric terminology with which pre-service student teachers enter the rural university. Even though it was found that the majority of the students had a fairly good knowledge of the geometric terminology, the students performed better in dealing with visually presented terminology items than verbally presented ones. This raised a concern that the majority of students were operating at the Visualisation level of Van Hiele's geometrical thinking. The study revealed that the matrics of 2014 performed better in all aspects tested than pre-2014 matriculants. It can be concluded that curriculum constraints due to the ongoing changes in the school mathematics curriculum might have adversely affected students' performance in geometry. The study also gives insight into the quality of students received by universities for teacher education courses which reflects the quality of geometry learning in our schools.

Visual and verbal representations in geometry should complement and supplement each other to enhance conceptual understanding. The use of multiple representations carefully built into the geometry curriculum will ensure that students meaningfully understand the concepts they are learning. Pre-service students and their educators need to adopt a combination approach since visual representations enhance spatial understanding and verbal representations promote mathematical terminology and mathematical language development besides general vocabulary and language development. The curriculum of the universities should include more opportunities for mathematics education students to familiarise themselves with school geometry content so as to allow them to teach it with understanding and meaning-making to learners in their careers as future teachers.

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Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

J.A. conducted the research and wrote the manuscript. K.M. made conceptual contributions and provided critical revision, guidance, editing and final approval of the document.

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