Pythagoras - Journal of the Association for Mathematics Education of South Africa
ISSN: (Online) 2223-7895, (Print) 1012-2346

Flexible teaching of mathematics word problems through multiple means of representation

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Dates:
Received: 21 Aug. 2020
Accepted: 03 June 2021
Published: 10 Aug. 2021

How to cite this article:
Moleko, M.M., & Mosimege, M.D. (2021). Flexible teaching of mathematics word problems through multiple means of representation. *Pythagoras, 42*(1), a575. https://doi.org/10.4102/pythagoras.v42i1.575

Flexible teaching of mathematics word problems is essential to improve learning. Flexible teaching is vital in terms of providing meaningful learning, creating inclusive learning spaces and making content accessible. As such, teachers need to strive to provide flexible teaching of mathematics word problems in order to optimise and maximise learning. In line with this notion, therefore, the qualitative case study reported in this article aimed to explore the implementation of one aspect of universal design for learning (UDL), namely multiple means of representation (MMR), to guide flexible teaching of mathematics word problems. Data were collected using focus group discussions, reflection and observation sessions in which five high school mathematics teachers and a Head of Department were involved. The teachers participated in a mini-workshop on the application of the UDL principles which was organised to introduce and induct them to the approach. The study showed that MMR can be used to help guide flexible teaching of mathematics word problems by providing varied options for comprehension: options for language, mathematical expressions and symbols, as well as options for perception. The findings of the study recommend the need for teachers to adapt their teaching by considering the application of the MMR principle to guide and promote flexible teaching of mathematics word problems.

Keywords: flexible teaching; mathematics word problems; teaching strategy; universal design for learning; multiple means of representation.

Introduction

Learners who face challenges in terms of English proficiency find mathematics word problems (MWPs) difficult to solve (Vula & Kurshimla, 2015). This is because MWPs require learners not only to know how to work with numbers, but also to possess other skills such as to identify, understand, interpret, create, communicate and compute numbers (UNESCO, 2005, p. 21) which are skills they often lack. Furthermore, learners have limited knowledge of mathematical vocabulary, which makes it difficult for them to understand and solve these problems. According to Seifi, Haghverdi and Azizmohamadi (2012), many teachers find MWPs challenging to understand and teach. As a result, they resort to teaching them in a 'mechanical' manner, which does not cultivate and deepen understanding (Sepeng & Madzorera, 2014). A mechanistic way of solving mathematical problems further causes learners to fail to develop personal connections and understanding between the mathematical concepts (Goldberg & Bush, 2003). According to Liljedah, Trigo, Malaspina and Bruder (2016), mechanical problem-solving refers to a method of solving mathematical problems by merely applying previously learned formulae to new similar situations. This way, true solutions can be reached by solving mathematical operations in a certain specific outlined order (Bal, 2015).

Although this seems to be the preferred way of solving mathematical problems, the challenge it poses is that it limits learners' critical thinking skills and through this method learners are unable to demonstrate the acquisition and ability to apply mathematical skills in new contexts as envisioned in the Curriculum Assessment Policy Statement (CAPS) document (DBE, 2011). Solving mathematical problems mechanically promotes a high level of engagement with routine problems by merely applying formulae or certain defined steps without necessarily understanding the underlying concepts behind the applied procedures. This form of problem-solving limits the learners' ability to solve and master the non-routine problems that are based on daily life and that guide learners in developing unique ways and strategies for problem-solving (Anderson, 2009; Elia, Van den Heuvel-Panhuizen, & Kolovou 2009; Sahid 2011). Learners who are exposed to this form of teaching practice often discount reality in their solution processes, thus generating conclusions that are mathematically correct but situationally incorrect or inaccurate (Webb & Sepeng, 2012).

Drawing from the above discussion, a deduction could be made that teachers who prefer mechanical ways of problem-solving do not make substantial efforts in terms of making their teaching flexible to ensure that all learners productively learn MWPs. On the basis of this, therefore, in this study we argue that MWPs could be taught productively through the application of the aspects of universal design for learning (UDL), which promotes the idea of flexible teaching through inclusive practices.

A consideration of inclusive practices is vital in terms of creating a learning environment where teachers could develop supportive relationships with learners and also increasing learner participation and engagement (Guðjónsdóttir & Óskarsdóttir, 2015). Teachers who apply inclusive teaching practices show flexibility in their teaching in that they apply different strategies that accommodate a broad range of learners in their classrooms (Engelbrecht, Nel, Nel, & Tlale, 2015; Ojageer, 2019). Such flexibility is essential and beneficial because it allows teachers to respond to different learner abilities, needs and interests (Murawski & Hughes, 2009). Teachers whose teaching is flexible make it easy for learners to follow precisely what they are trying to teach them. Furthermore, teachers who apply flexible approaches are able to increase learner participation and engagement (Hennessy, Deaney, Ruthven, & Winterbottom, 2007) and this practice is important in ensuring that no learner is left behind (Hill, 2007). Other ways in which teachers could demonstrate flexibility and responsiveness include differentiating the instruction to address individual learner misunderstandings, building on learners' interests, etc. (Rock, Gregg, Ellis, & Gable, 2008). Our belief based on all this is that teachers who are flexible in their teaching carry the idea that their plans can change very swiftly, sometimes with notice and sometimes without. Such an idea thus necessitates teachers to be proactive in terms of planning their teaching by considering various teaching strategies to accommodate the different learning styles. It also cultivates the culture of anticipating elements of diversity, which may necessitate adaptation of teaching strategies. In line with this, literature indicates that UDL can be used as a front-loader (Cooper-Martin & Wolanin, 2014). This means that the teacher has to incorporate the UDL strategies during the creation of instruction and assessments, instead of adjusting lessons or assessments afterwards. It is against this backdrop that this study aims to explore the implementation of UDL to guide flexible teaching of MWPs.

Literature review

Universal design for learning is 'an approach to teaching that consists of the proactive design and use of inclusive instructional strategies that benefit a broad range of learners including students with disabilities' (Scott, McGuire & Embry, 2002, p. 2). This framework for teaching was coined to promote multiple teaching practices and integration of the current best approaches to engage learners to address their different learning styles. Therefore, UDL makes it possible for teachers to meet the learning needs through a fusion of teaching approaches, designs and technologies. Dalton, Mckenzie and Kahonde (2012) note that UDL is a teaching strategy used to accomplish the broader goal of inclusive education. On the other hand, Van Jaarsveldt and Ndeya-Ndereya (2015) state that UDL is the most appropriate teaching strategy to address diversity within the classrooms and to promote flexible teaching that is all-encompassing. According to Burgstahler (2008), UDL is intended to maximise learning and to inculcate the culture of flexible and inclusive teaching practices (Scott et al., 2002).

Universal design for learning inspires the teachers to anticipate, embrace and acknowledge diversity within the classroom. As such, teachers are expected to plan their lessons to address the diverse needs of learners from the inception of their teaching rather than to wait until teaching has taken place (Israel, Ribuffo, & Smith, 2014). This therefore means that UDL requires teachers to be proactive rather than reactive, in terms of addressing the needs of the learners, thus calling for teaching practice adaptation.

Mathematics word problems

A MWP denotes text that describes a situation assumed to be familiar to the reader and poses a quantitative question that subsequently requires an answer to be derived through mathematical operations performed on the data provided in the text form, or otherwise inferred (Greer, Verscheffel & De Corte, 2002). According to Kasule and Mapolelo (2013) MWPs' content is presented in the form of stories. Palm (2009) refers to MWPs as representations of real-life situations.

Texts of MWPs are stretches of amalgamated forms (e.g. clauses or sentences), written in a particular dialect code (e.g. the English language) and register (e.g. mathematical terminology), and with a distinctive internal organisation (i.e. a textual structure) that can be logically and rationally understood by readers who bring with them expectations, interests, viewpoints, interpretations and prior reading experiences (Oliveira et al., 2015).

Mathematics word problems are an essential part of the mathematics curriculum because of their ability to promote realistic mathematical modelling and problem-solving (Van den Heuvel-Panhuizen & Drijvers, 2014). These problems empower learners to realise the connections between real-life situations and their classroom mathematical knowledge (Sepeng & Madzorera, 2014).

Although a significant role of the MWP is undisputable in terms of promoting realistic mathematical modelling and developing personal connections between real life and classroom mathematical knowledge, this mathematical genre has proven to be challenging for most learners and teachers alike (Seifi et al., 2012). Teachers usually teach MWPs in a mechanistic manner because they often find the teaching of these problems difficult (Sepeng & Madzorera, 2014). Mechanistic teaching hampers problem-solving and this is often seen through learners who discount reality in their solution processes, thus generating conclusions that are mathematically correct but situationally incorrect or inaccurate (Sepeng 2010; Webb & Sepeng, 2012).

The teaching of MWPs is further complicated by necessitating the learners' ability to recognise, comprehend, construe, construct, communicate and work out numbers, which are skills most learners seem not to have (Unesco, 2005). The teaching of word problems is made more intricate by the learners' inability to read (Gooding, 2009), which is evidenced by being unable to determine missing information, generate number sentences and set up calculation problems (Fuchs et al., 2008). Huang and Normandia (2008) state that most learners commit more mistakes when solving word problems than when solving equivalent number problems and this is because word problems demand strong mathematical calculations along with other types of knowledge such as linguistic knowledge and analysis which are the skills that most teachers do not cultivate in learners (Sepeng, 2013). On the other hand, the teaching of word problems proves to be more challenging especially in teaching and learning settings wherein English is a medium of instruction and a second language of the learners (Essien, 2013). According to Barwell (2009), such teaching and learning settings present teachers with challenges and extra demands for them to pay attention to mathematics, pay attention to English (the language of learning and teaching) and also pay attention to mathematical language and register. Adding to the challenge of teaching MWPs is the fact that teachers themselves also find word problems difficult to solve (Seifi et al., 2012).

Deducing from the above, it is reasonable to note that MWPs are a difficult genre to teach to learners. It is due to their complex nature that Webb, Campbell, Schwartz and Sechrest (1966) label them as 'demon' problems. The fact that they are often taught in a mechanistic manner, which does not cultivate understanding, shows that substantial efforts in terms of applying flexible and inclusive practices to cater for different learning styles are not made. It is against this backdrop that this study is purposed to explore the implementation of UDL to guide flexible teaching of MWPs.

Universal design for learning

Universal design for learning principles (which make up the UDL framework for teaching) were employed in this study to guide flexible teaching of MWPs. The UDL framework is a broader framework that comprises three principles, namely multiple means of representation (MMR), multiple means of action and expression (MMAE) and multiple means of engagement (MME) (Center for Applied Special Technology [CAST], 2011). These three principles are linked to the three brain networks, namely the recognition, strategic and affective networks (Grabinger, Aplin, & Ponnappa-Brenner 2008). The recognition network (MMR) addresses the 'what of learning', while the strategic network addresses the 'how of learning' and the affective network addresses the 'why of learning' (Rose & Meyer, 2006). According to neuroscientists, the recognition network (MMR) makes it possible for learners to receive and analyse information, the strategic network (MMAE) makes it possible for learners to generate patterns and develop strategies for action and problem-solving, while the affective network (MME) helps fuel motivation and guide the ability to establish priorities, focus attention and choose action (CAST, 2011). In line with this, the CAST team formulated a comprehensive framework to serve as a guide towards teaching that is flexible and inclusive (see Table 1).

This study thus considers some of the aspects of this framework in order to analyse the teaching of MWPs and to provide some guidelines in terms of applying flexible teaching methods. In order to make sense of the data, the emerging themes were organised according to the UDL principles (i.e. MMR, MMAE and MME) and their respective sub-themes.

TABLE 1: Three principles of universal design for learning.

Provide multiple means of engagement	**Provide multiple means of action and expression**	**Provide multiple means of representation**
Purposeful, motivated learners	***Strategic and goal-oriented learners***	***Resourceful and knowledgeable learners***
Provide options for self-regulation	**Provide options for comprehension**	**Provide options for executive functions**
+ Promote expectations and beliefs that optimise motivation	+ Activate or supply background knowledge	+ Guide appropriate goal setting
+ Facilitate personal coping skills and strategies	+ Highlight patterns, critical features, big ideas, and relationships	+ Support planning and strategy development
+ Develop self-assessment and reflection	+ Guide information processing, visualisation, and manipulation	+ Enhance capacity for monitoring progress
	+ Maximise transfer and generalisation	
Provide options for sustaining effort and persistence	**Provide options for language, mathematical expressions, and symbols**	**Provide options for expression and communication**
+ Heighten salience of goals and objectives	+ Clarify vocabulary and symbols	+ Use multiple media for communication
+ Vary demands and resources to optimise challenge	+ Clarify syntax and structure	+ Use multiple tools for construction and composition
+ Foster collaboration and community	+ Support decoding of text, mathematical notation, and symbols	+ Build fluencies with graduated levels of support for practice and performance
+ Increase mastery-oriented feedback	+ Promote understanding across languages	
	+ Illustrate through multiple media	
Provide options for recruiting interest	**Provide options for perception**	**Provide options for physical action**
+ Optimise individual choice and autonomy	+ Offer ways of customising the display of information	+ Vary the methods for response and navigation
+ Optimise relevance, value, and authenticity	+ Offer alternatives for auditory information	+ Optimise access to tools and assistive technologies
+ Minimise threats and distractions	+ Offer alternatives for visual information	

Source: CAST. (2011). *Universal design for learning guidelines version 2.0.* Wakefield, MA: Author.

Theoretical framework

Critical emancipatory research (CER), which borrows its roots from social constructivism (Nkoane, 2012), was adopted to guide this study. In the same way as social constructivism, CER promotes the notion that knowledge is a product of social interaction. This framework thus espouses the notion of people working together to construct knowledge (Tlali, 2013). The framework also promotes the notion of space creation for people to share ideas and knowledge with an intent to explore an issue of interest from manifold angles (Tsotetsi, 2013). According to Campanella (2009), CER requires people involved in the research project to be viewed as 'capable speaking beings' and not just objects that cannot think or do anything for themselves. It also advocates for the inclusion of all the people including the marginalised to identify solutions to their own challenges (Mahlomaholo, 2009). On the basis of this therefore we regard CER as a lens that recognises the 'silent' and 'silenced' voices and as such affords all people including the marginalised opportunities to engage in issues of their concern, deliberating them from their point of view as informed by their lived experiences. Researchers who apply this theoretical framework believe that knowledge is a key tool that should be turned into practice that transforms the situation and empowers the people (Al Riyami, 2015). Deducing from the goal of CER, it is reasonable to indicate that it (CER) advances the agenda of human empowerment, transformation and liberation for better living or functioning.

Research methods and design

Research design

In order to gather data significant to the teaching of MWPs, in particular how the teachers teach this mathematical genre in line with UDL's multiple means of representation, an exploratory descriptive design was adopted. The exploratory descriptive design is usually used when there is limited existing information available on a topic in order to gain new insights and to understand the phenomena (Grove, Burns & Gray, 2013). In line with this therefore an exploratory descriptive design was adopted in this study because there is limited existing information regarding the issue under investigation (i.e. the implementation of MMR to guide flexible teaching of MWPs). Furthermore, exploratory descriptive design was deemed flexible and appropriate in terms of gathering data that would help address the research question of this study: how can the multiple means of representation be implemented to guide flexible teaching of mathematics word problems?

Sample - Selection of the participants

Purposive sampling, which is an informant selection tool that is widely used in qualitative research, was used in this study (Tongco, 2007). The purposive sampling technique refers to the deliberate choice of participants due to the qualities they possesses (Etikan, Musa & Alkassim, 2016). When this technique is used, the researcher usually decides what needs to be known and determines the people who can and are willing to provide the information by virtue of knowledge or experience (Bernard, 2002). This technique was selected in order to help the researcher meet or fulfil a specific purpose (i.e. to explore the implementation of UDL in order to guide flexible teaching of MWPs) (Naderifar, Goli, & Ghaljaie, 2017). As such, five high school mathematics teachers including the head of the mathematics department in one school in the Thabo-Mofutsanyana district in the Free State were selected to participate in the study. These teachers had more than 10 years of experience in the teaching of mathematics. Besides the fact that these teachers were selected because they were teaching mathematics, their selection was also motivated by the fact that they had the necessary background to understand and teach these types of problems, which is the impression they gave during the first meeting when the researcher explained the rationale for conducting the study and also highlighted what she deemed to be the problem. They all expressed that they are familiar with this type of problem and they further alluded that these problems (MWPs) were not only difficult to teach but also that the learners found them difficult to understand and solve.

Mini-workshop training on the principles of universal design for learning

The teachers who were selected to participate in the study took part in the mini-workshop training on the application of UDL principles. These teachers participated in the activities that involved the application of the three UDL principles, namely MMR, MMAE and MME. They later planned their lessons on the teaching of word problems in line with these principles. During the observation sessions, the UDL guideline 2.0 (see Table 1) was used as a tool to evaluate how they implemented the principles, and the gathered data were later analysed to indicate how the principles (MMR specifically for the purpose of this article) were implemented.

Teaching mathematics word problems

During the teaching of MWPs, the researchers sat in class to observe how the teaching was carried out. The sessions were both audio and video recorded. Focus was placed more on how the teaching was carried out in order to later recommend the appropriate and more flexible teaching strategies that could be applied to teach this mathematical genre. The UDL guideline 2.0 was used as a guide or point of reference to assess the teaching (CAST, 2011).

Data collection and analysis

Focus group discussion

The focus group comprised five high school mathematics teachers. The teachers were provided with opportunities to clarify and discuss their teaching practices during the focus group discussions.

Observations

Qualitative observations are the types of observations in which the researcher records field notes on the behaviour and activities of individuals at the research site (Creswell, 2009). According to Gibson and Brown (2009) observational research can be conducted for a number of reasons; however, it is usually a part of a general interest in understanding, for one reason or another, what people do and why. In this study, the structured observation schedule was administered in four classrooms for three consecutive days. The researchers observed the sessions following the UDL guidelines, which were to provide a guide for interpretation of how the word problems were taught. The classroom observation schedule focused on the following:

1. the MMR used
2. the consideration of MMAE
3. the application of MME.

The three aspects above were focused upon in order to provide guidance in terms of multiple ways in which the content could be represented, multiple ways in which learners could be provided with options to demonstrate their learning processes, as well as various ways in which learners could be taught how to use the available formats, tools and technology to learn word problems.

Reflection sessions

Reflection sessions were conducted in order for the teachers to reflect upon their teaching practices. According to Magalhães and Celani (2005) reflection is a form of practice that involves, among others, the discussion of the aspects that were previously ignored, rethinking the situation and attributing newly generated meanings to situations already discussed. Reflections therefore enable the participants to think critically about the issue, and thus give meaning to the experiences. In this study, the reflective sessions served as platforms where teachers shared their experiences in terms of teaching word problems as well as highlighting the strategies they used. These sessions were conducted after the teachers had given the lessons.

Findings and discussion

This section reports on the findings and discussion, and recommendations. For the purpose of this article we discuss the findings that are related to the MMR. The results for the MME and MMAE are not reported in this article but are reported elsewhere. We therefore focused on the aspects of UDL (see Table 1) that make up this principle (MMR) in order to draw the findings and make sense of the generated data. The data reported in the subsequent sections emerged from the observations, reflection sessions and focus group discussions. Teachers were observed while teaching the different topics involving MWPs. The UDL framework as shown in Table 1 was used to analyse and comment on the teaching of MWPs. Teachers' experiences in terms of teaching MWPs were also narrated through the reflection sessions and focus group discussions. The data that emerged from the three instruments revealed some of the best practices in terms of teaching MWPs in line with the MMR principle.

Multiple means of representation

The following sections provide an analysis of data in line with the UDL principle of MMR. The examples that are discussed in the subsequent sections were chosen because they are word problems and the explanations provided, in terms of how they were carried out, show the operationalisation of the MMR principle.

Explanation

> Teacher 2: In order to give the next three terms you need to first check the relationship between the terms. For example, two multiply by four is equal to eight. Eight multiply by four is equal to thirty-two. Again eight divide by two is equal to four and thirty-two divide eight is equal to four. When you check this, you will realise that four is a common number, which you either have to multiply the current number with it in order to get the next term or divide the next term with it in order to get the previous term. Remember when we dealt with the relationships between the operational signs, I showed you how multiplication and division relate.

The word problem given in Episode 1 is a geometric series and this can easily be identified when the series is represented numerically as '2; 8; 32; …'. In order to respond to the three questions posed, the teacher had to address a few aspects in her teaching. In her explanation above the teacher first determined the 'common' number, which could be multiplied with the current term in order to yield the next term. The teacher also showed the learners that the same 'common' number can be obtained if the next term is divided by the previous term. Thus the teacher '*highlighted patterns*' as one way of 'providing options for comprehension' according to the UDL MMR principle, thus enabling learners to represent the series correctly and to make generalisations. The *critical features* of the series were also highlighted by the teacher in line with this principle in order to help the learners solve the problem and establish the general formula. The teacher also indicated the *big ideas* regarding the given problem, thus *showing the relationships* among the terms as well as the operational signs used. According to CAST (2011) teachers

BOX 1: Episode 1.

Context: [the lesson on word problems was conducted based on the topic: 'Sequence and Series' during the observation session and the example below was taught to the learners].

Example: Consider the following sequence/series and answer the subsequent questions:

Busisiwe gave two marbles to Pule, eight to Lucky and thirty-two marbles to Moreki. If she has to continue giving marbles in this manner, how many marbles will she have to give to Thabo, Tseko and Mandla respectively?:

1. Represent this sequence/series numerically
2. Write down the general formula to represent this sequence/series if the last term is 32768
3. Write the general formula for the nth term

BOX 2: Episode 2.

Context: [the teacher reflected on the learners' struggle to convert the 'word problem expression' into simple algebraic expression during a reflection session]
Solve the following expression:
'five is less than three less than the number x'

BOX 3: Episode 3.

Context: [The teacher discussed the strategy that he used to demonstrate the graph shift during a focus group discussion through the use of the example below]
Read the word problem below and answer the questions that follow:
The triangular pyramid, which Dimpho built, has four layers namely: layer one, layer two, layer three and layer four. These layers are arranged such that they reflect the squares of the numbers of the layers they represent respectively. Dimpho later added two more triangles to each layer of the pyramid that she had already built to form a new triangular pyramid. She thus ended with the new triangular pyramid, which reflects four layers that represent the squares of the numbers: one, two, three and four plus two more triangles added to each layer: i. Represent algebraically the equations that are described from the above mathematics word problem. ii. Draw the two graphs on one set of Cartesian plane and describe the graph shift that has occurred.

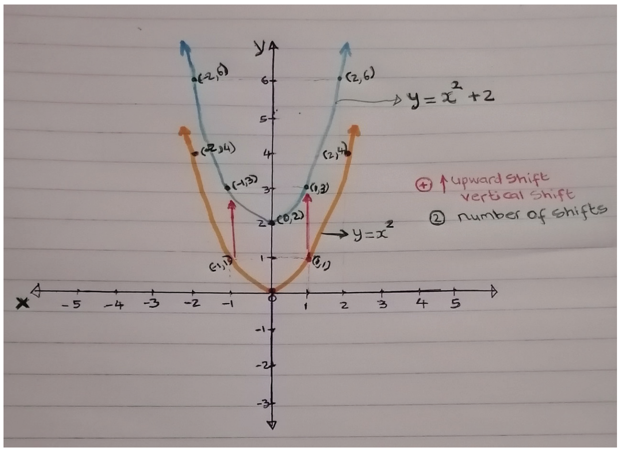

FIGURE 1: Shifting of the graphs.

who consider these forms of practices provide options for comprehension which is necessary to assist learners receive and analyse the word problems.

Explanation

> Teacher 3: Learners usually find it difficult to represent the two expressions that are reflected in this problem. In order to simplify it and make it understandable to all of them, I separate the two expressions and work them out separately.
>
> Five is less than – part 1, i.e. 5 <
>
> Three less a number – part 2, i.e. x – 3 not 3 – x

According to Teacher 3, learners find it difficult to solve and make a representation of the given problem (Episode 2) because they fail to realise that there are two expressions embedded in it. Learners also found this problem difficult to solve because they did not know which signs to use in order to represent and solve it. In order to simplify this problem the teacher broke down the problem into two simple comprehensible expressions namely: *Five is less than* – part 1, i.e. 5 <, and *Three less than a number* – part 2, i.e. $x - 3$ (not $3 - x$). This way the teacher *guided information processing*. According to the teacher, learners understand the smaller parts of a given complex word problem and are able to make connections that lead to understanding the problem holistically rather than holistically solve the problem from the beginning. According to Mevarech and Kramarski (1997) guiding information processing enables leaners to be aware of problem-solving and this practice induces learners to activate the four-phase problem-solving model suggested by Pólya (1973).

Explanation

> Teacher 5: I find the use of different colours to be useful especially when I teach about the shifting of the graphs.

According to Teacher 5, demonstration of the shifting of the graphs can better be facilitated through the use of different colours (see Figure 1). Consequently, the use of different colours serves as a visualisation enhancing tool that promotes the visualisation skill, which is one of the vital skills necessary for mastering word problems. The teacher's strategy for using different colours is supported by Shabiralyani, Hasan, Hamad and Iqbal (2015) who further stress that the use of different colours makes visual aids perceptible to the entire class. The different colours thus assist in providing a clear contrast and also making the plotted graphs easily visible.

Through the use of diagrams and accurately measured graph sheets, learners were able to see the shift that has taken place between the two given equations and this improved their problem-solving (Kashefi, Alias, Kahar, Buhari, & Othman, 2015). Such instruments thus increased the visual impact, interactivity and spontaneity which according to Williams (2004) provides high-quality learning experiences and also improves learner focus.

The example provided by the teacher shows the importance of customising information in different formats to maximise learning. The teacher's example also confirms that visualisation is a skill, which teachers can nurture in learners through the use of different representations (e.g. diagrams, number lines) (Alex & Mammen, 2018; Gilbert & Auber, 2010). Courtad (2019) also supports the notion of multiple representation in that it improves learner perception, helps decode language and symbols, and reinforces understanding.

The data reported in this section outline what the teachers considered to be important in terms of providing options for language, mathematical expression and symbols. The data further suggest the need for teachers to vary their teaching and use the appropriate resources to optimise teaching.

In terms of addressing the mathematical vocabulary, the teachers commented as follows:

> 'It helps really to address vocab in your teaching. Sometimes these learners fail to solve mathematical problems because of some of the words they do not know.' (Teacher 1)

> 'That is so true because some of these terms that they are using, they also come across them in other subjects and if not clarified, they bring about confusion. For example a term such as "function" can cause confusion if not thoroughly explained.' (Teacher 4)

The teachers indicated the importance of clarifying the mathematical vocabulary since it carries meaning in the given problems. According to Teacher 1, these mathematical key terms are important to teach (Riccomini, Smith, Hughes & Fries, 2015) and to clarify because they carry meaning and directive in terms of what should be solved. As for Teacher 4, clarification of these terms is important especially when it is linked to application of such similar terms in other 'subjects' as opposed to their application in mathematics in order to draw a clear distinction (Owens, 2006; Widdows, 2003). For example, the term 'function' is used within the mathematical context to refer to an equation, which denotes the 'input, process and output' concept. However, in Life Sciences, the same term 'function' describes the work of a specific organ in the body. The teaching of mathematical vocabulary further helps teachers to expose learners to terms that can be used 'interchangeably' (Godino, 1996) (e.g. yearly or annually, altogether or sum, remainder or difference, etc.). Chitera (2009) also recommends the practice of explicitly teaching mathematical vocabulary to help learners gain control over mathematical language, which is important for them to comprehend and master the MWPs.

It can be deduced from the example provided that a word in English may not necessarily be interpreted, explained and applied the same way as it is interpreted, explained and applied in mathematics (Kashgary, 2011), which is why the teachers have to address this in their teaching to help eliminate confusion, thus promoting understanding language across disciplines. Explicit teaching of mathematical vocabulary is also supported by literature, which indicates a strong correlation between mathematical vocabulary and comprehension of mathematical content (Monroe & Orme, 2002):

> 'If we look at the example that was provided earlier … eeehhh that one of: "Five is less than three less than a number". Learners usually read it literally as it is and they translate it into algebraic expression written as 5 < 3 < x which is incorrect. So when teaching a problem like this to learners I, break it down into simple expressions and then translate it into algebraic expressions that can easily be solved.' (Teacher 1)

> 'Breaking this word problem into simple expressions helps, for example "three less than a number"; you are able to indicate that it should be represented as x – 3 and not as 3 – x.' (Teacher 5)

The above extracts indicate the significance of teaching mathematical language and structure. In order to achieve that, Teacher 5 highlighted the need for teachers to break the complex word problems into simple expressions in order to correctly represent them algebraically. As such, teachers should emphasise in their teaching the fact that learners should not '*literally*' read the word problems and use left to right translations; rather they should read them carefully and try to understand them holistically. For example: '*Five is less than three less than a number*'. This word problem can be represented algebraically as: $5 < x - 3$. In this case, the expression 'is less than' denotes '<' and the expression 'less than' denotes '–'. Looking at the expression 'three less than a number', the correct representation of the expressions is $x - 3$ and not $3 - x$ as it reads. In order for this expression to be correctly represented, careful reading, holistic understanding and knowledge of the mathematical structure and syntax are necessary. This will enable learners to realise that three less than a number is not $3 - x$ but $x - 3$. The holistic understanding of the problem helps to ensure the correct 'placement or location' of the variable (x) and the number (3).

Deducing from the teachers' explanations, it is notable that the teaching of mathematical structure and syntax helps to translate texts into correct variable representations, which subsequently enable learners to solve the word problems. In order to clarify the syntax and structure, the teachers need to engage learners in reading, illustrate the representations of the word problems and show the application of the correct signs or symbols in the given problems. This way, the learners will realise that word problems are not necessarily represented in writing the same way as they are read and that the signs or symbols to be used depend on what the problem denotes or insinuates:

> 'Breaking down word problems helps to show learners how to write text in mathematical form. If learners read the whole word problem, it may be difficult for them to even realise that three less than a number should be represented as x – 3 and not 3 – x.' (Teacher 2)

> 'To justify or clarify why the expression "three less than a number" is written as x – 3 and not 3 – x one can use money concept … t.' (Teacher 4)

According to Teacher 2, breaking down the word problems is one strategy that teachers can use to help learners with *decoding of text* and to convert such text into correct algebraic expressions with *mathematical notations and symbols*. Since left to right translation does not always yield the correct interpretation and representation of text into variables, it is important for teachers to use various forms of explaining such representations. For example, in order to explain the swapping around of the number and the variable (e.g. $3 - x$ to $x - 3$) Teacher 4 suggested the use of the 'money concept'. The use of the money concept thus serves as another form of representation or medium through which the teacher could reinforce the correct mathematical representation of the mathematical expression. Breaking down of the complex word problem into smaller comprehensible expressions also assists in terms of distinguishing the expressions (e.g. $x - 3$ versus $3 - x$), thus promoting the correct interpretation, representation and decoding of text into mathematical notation and symbols:

> 'What does "at least" mean?' (Teacher 4)

> '*e bolela bonyane* [it means little].' (Learner 1)

> 'I think it refers to something smaller.' (Learner 2)

> 'How would you represent this expression symbolically?' (Teacher 4)
>
> 'I will represent it as "≤".' (Learner 3)

Through lesson observation, the significance of promoting understanding across different languages when teaching mathematical concepts was notable. The learners were given the expression 'at least'. Mathematically, the expression denotes 'greater than or equal to' represented as '≥' symbolically. However, during the lesson observation, learners seemed to have difficulty working out the word problem that contained that expression. This is because the expression was interpreted and understood by learners differently, according to various contexts and thus they used their backgrounds to attach meaning to the expression. For example, 'bonyane' in Sesotho, 'buncinci' in isiXhosa, and 'okungenani' in isiZulu. The teacher seemed to be aware of this challenge, which is why she firstly asked learners what their understanding of the expression '*at least*' is, thus *eliciting their pre-knowledge*. Based on the answers they provided, the teacher realised that the expression was not understood within the mathematical context and that, as a result, learners used the incorrect mathematical sign to express it. The teacher then explained to learners that the expression does not refer to 'less than' or 'small' even though that may have been insinuated in their languages. The teacher further indicated that mathematically, the expression 'at least' denotes 'greater than or equal to' and does not refer to minimum as in ordinary English. From the lesson presentation and how the expression was explained, it became clear that there is a need for teachers to *promote understanding across different languages* and to explain mathematical expressions such that the meanings carried or embedded within the home language contexts do not become barriers towards learning and solving MWPs. As such, the expression ought to be defined and explained correctly in context, so that learners do not misinterpret it and thus apply the incorrect mathematical symbols. Nkambule (2009) also supports the issue of promoting understanding across different languages in order to reinforce understanding of mathematical concepts.

During the reflective session, the teachers indicated the need to customise the display of information in order to provide options for perceptionsm offer alternatives auditory and visual information:

> 'It helps most of the time to give word problems and the picture alongside to enable learners understand the word problem.' (Teacher 3)
>
> 'That is what I do when I teach them. I make some drawing representations in order to explain the concepts.' (Teacher 4)

Teacher 3 noted the significance of providing pictures alongside the word problems as another form of representation to aid learners' comprehension of word problems. This form of customising the display of information helps learners to understand and conceptualise the word problem. Although adding a picture may cater more for visual learners, other learners with different learning styles may also benefit from this practice. The expression '*It helps most of the time*' stresses the significance of such practice that should be cultivated when word problems are administered. According to Teacher 4 this practice is not only supposed to be adopted when the learners are given assessments; however, the teachers should also apply it also in their teaching. For example: learners may be given a mathematical problem in the form of text. A picture that highlights some of the features mentioned in the text may be provided alongside the word problem. This will promote visualisation of the problem and assist learners to realise what the problem is all about and what it requires to be solved. This converges learner thinking in the 'right direction'.

It can therefore be deduced from what the teachers noted that some learners swiftly grasp content if information is presented to them in multiple formats. Thus, learning and transfer take place with ease when multiple representations are provided because such representations allow learners to make connections that are necessary for them to master the word problems.

Conclusion

This article explored how MMR can be implemented to guide flexible teaching of MWPs. The study demonstrated that the MMR principle provides a flexible and a comprehensive framework for the analysis of teachers' practice. This principle further encourages the consideration and balancing of multiple processes and orientations in teaching, rather than a more restricted focus. The application of MMR in the teaching of word problems thus serves as a positive contribution to the field, by promoting, among others, work to synthesise different perspectives and develop a more holistic view, in both teaching and research.

The MMR principle constitutes three themes, namely providing options for comprehension, providing options for language, mathematical expressions and symbols, and providing options for perception. The study thus indicated the correlation among these three themes in terms of formulating MMR. For instance, the findings of the study indicate that in order for teachers to make it possible for learners to receive and analyse information, they (teachers) have to provide learners with varied options for comprehension, options for language, mathematical expressions and symbols, and options for perception. Providing varied options is deemed important because learners differ in terms of how they receive, analyse and assimilate information. The study thus makes a contribution through lifting out and balancing these different themes within the MMR formulation by demonstrating the implications and significance of reinforcing comprehension of mathematical concepts, which is facilitated by the correct and appropriate use of language, mathematical expression and symbols, to make it possible for the content to be perceptible. The application of MMR also contributes towards shaping learners who are knowledgeable and resourceful (CAST, 2011), which is one of the educational goals that teachers should strive to achieve.

In line with MMR principle of UDL, findings indicate that the productive teaching of MWPs could be achieved by providing alternatives for comprehension, language, mathematical expressions and symbols, as well as providing alternatives for perception. The need for this principle to be applied is spelled out in the South African CAPS and its application is supported in order to help learners receive and analyse information. The application of this principle contributes in developing learners who are resourceful and knowledgeable. Although this principle plays such a pivotal role in guiding flexible teaching of MWPs, the CAPS does not specify how this principle can be applied, thereby leaving this task to the discretion of the teachers. However, in terms of applying the MMR principle when teaching word problems in this article, the following were regarded as good practices: highlighting patterns in a given word problem, outlining critical features of the given word problem, as well as the big ideas regarding the concept that is dealt with. Elucidating mathematical vocabulary and symbols, teaching and clarifying mathematical vocabulary, syntax and structure as well as teaching learners how to decode text, mathematical notation and symbols were also regarded as good practices in terms of guiding flexible teaching of word problems. The findings of this study thus indicate MMR as a promising strategy to help guide flexible teaching of MWPs. The teachers thus need to make their teaching flexible by considering this principle when teaching MWPs, thus providing choices for comprehension, choices for language, mathematical expressions and symbols, as well as providing options for perception.

Acknowledgements

Competing interests

The authors declare that there are no competing interests in the production or publication of this article.

Authors' contributions

This article is based on M.M.M.'s doctoral study, so M.M.M. collected and analysed the data used in the study. M.D.M. worked with M.M.M. to do further analysis, refinement and reshaping of the work included in this article, and finalised and submitted the article on behalf of M.M.M. and M.D.M.

Ethical considerations

The Ethical Clearance was obtained from the University of the Free State and the Clearance Certificate was obtained on 14-Mar-2017 (UFS-HSD2016/1194).

Funding information

The study on which this article is based was funded by the National Research Foundation (NRF).

Data availability

Data sharing is not applicable in this article as no new data were created or analysed in this article.

Disclaimer

The views and opinions expressed in this article are those of the authors and not an official position of the University of the Free State.

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