Design principles to consider when student teachers are expected to learn mathematical modelling

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This article sets out design principles to consider when student mathematics teachers are expected to learn mathematical modelling during their formal education. Blum and Leiß's modelling cycle provided the theoretical framework to explain the modelling process. Learning to teach mathematical modelling, and learning to solve modelling tasks, while simultaneously fostering positive attitudes, is not easy to achieve. The inclusion of real-life examples and applications is regarded as an essential component in mathematics curricula worldwide, but it largely depends on mathematics teachers who are well prepared to teach modelling. The cyclic process of design-based research was implemented to identify key elements that ought to be considered when mathematical modelling is incorporated in formal education. Fifty-five third-year student teachers from a public university in South Africa participated in the study. Three phases were implemented, focusing firstly on relevance (guided by a needs analysis), secondly on consistency and practicality via the design and implementation of two iterations, and lastly on effectiveness by means of reflective analysis and evaluation. Mixed data were collected via a selection of qualitative instruments, and the Attitudes Towards Mathematical Modelling Inventory. Through content analyses students' progress was monitored. Results analysed through SPSS showed significant positive changes in their enjoyment and motivation towards mathematical modelling. Student teachers require sufficient resources and opportunities through their formal education to participate regularly in mathematical modelling activities, to develop competence in solving modelling tasks, and to augment positive attitudes. This study adds value to the global discussion related to teachers' professional development regarding mathematical modelling.

Keywords: attitudes towards mathematical modelling; design-based research; design principles; formal education; learning mathematical modelling; mathematical modelling; professional development; student mathematics teachers

Introduction

Student mathematics teachers, who will also teach mathematical modelling in their future professional role as teachers, should purposefully and strategically be prepared for this task because both teachers and students find the topic cognitively demanding. A well-prepared teacher should include real-life examples and applications in mathematics teaching as an essential component in mathematics curricula to develop problem-solving and cognitive abilities in learners (Department of Basic Education, 2011), but also to help learners to better understand the world, to support mathematics learning (e.g. concept formation), to develop various mathematical competencies and appropriate attitudes, and to contribute to an adequate picture of mathematics (see Blum, 2015; Wessels, 2017).

Usually with application questions the focus is to link a mathematical topic to reality, but with real-life questions the focus is to link reality to mathematical topics. The difference in focus is explained through posing the following questions (Stillman, Galbraith, Brown, & Edwards, 2007): *Where can I use this particular piece of mathematical knowledge*, as opposed to, *Where can I find some mathematics to help me with this problem*? It is possible for teachers to learn both modelling and the teaching of modelling. Borromeo Ferri (2018) has emphasised the necessity of trainee mathematics teachers to have vast opportunities to deal with modelling activities on a theoretical and a practical level. Biccard (2019) confirmed that teachers should be given opportunities to experience tasks in the role both of learner and teacher. One such possibility is to follow a design-based research methodology (compare Biccard, 2019).

Thus, two primary directives provided a stimulus for this study. Firstly, student teachers require adequate preparation in respect of their 'knowledge-in-action' of mathematical modelling, while also fostering positive attitudes towards this topic. This 'knowledge-in-action' includes competencies both as modellers themselves and as teachers of modelling activities (see Blum, 2015; Durandt & Lautenbach, 2020; Ng, 2013), and experiences that provide a stimulus for growth in attitudinal aspects such as motivation, value, self-confidence and enjoyment of mathematical modelling (Chamberlin, 2019; Jacobs & Durandt, 2017). Secondly, mathematical modelling has a positive influence on the teaching and learning of mathematics and is included in the current South African mathematics curriculum (Department of Basic Education, 2011). These two directives are not only limited to the South African context but are also relevant for other contexts where the professional development of future teachers is well established and mathematical modelling included in curriculum documents (Anhalt & Cortez, 2016; Blum & Borromeo Ferri, 2016; Tan & Ang, 2013).

The aim of the study reported in this article was to identify suitable design principles by exposing student teachers (both as modellers and as teachers) to a well-planned set of mathematical modelling activities, while monitoring their development in competencies and their change in attitudes over time.

The research questions informing this study were:

1. What design principles can be identified to prepare student teachers for facilitating mathematical modelling activities?
2. What shortcomings and progression in student teachers' mathematical modelling competencies can be identified during the intervention?
3. What change in student teachers' attitudes towards mathematical modelling activities can be identified during the intervention?

Awareness of the ways mathematical modelling can be learnt through incorporating a selection of key elements and fundamental principles in their formal education could contribute to decreasing the cognitive burden of student teachers and cultivating positive experiences. The article will report on relevant theoretical aspects, the design-based strategy (organised through different phases) that formed the situational context to provide an opportunity for student teachers to learn modelling, as well as the results and findings from mixed data explaining their shortcomings, progression and experiences, ending with concluding remarks.

Conceptual framework

The intention is this study was to identify key elements and fundamental principles to guide the integration of mathematical modelling in the formal education of student mathematics teachers in a purposeful and strategic way. Two underlining theoretical perspectives are relevant: (1) the mathematical modelling process and the competencies needed for doing mathematical modelling, and (2) key characteristics of a design-based study.

Learning mathematical modelling

Mathematical modelling is a cyclic process that describes the translation between reality and mathematics in both directions. This process consists of certain sub-processes as shown, in an ideal-typical form, in Figure 1 (the modelling cycle from Blum & Leiß, 2007).

The sub-processes are to construct a situation model from a real-world problem, to generate real-world facts, data and relations, to simplify and structure the data, to mathematise (represent the data mathematically), to work within mathematics, and finally to interpret and to validate the mathematical results with respect to the real-world situation. A variety of illustrations of the modelling process exist in the literature (e.g. Stillman, Kaiser, & Lampen, 2020). The advantage of the illustration from Blum and Leiß (2007) is that individual steps (1–7) separate the phases of a typical mathematical modelling process. Students usually do not follow those steps in linear order when solving modelling tasks, but often 'bounce' between them. Each step can potentially be a cognitive barrier for students (Blum, 2011, 2015; Stillman, 2019; Stillman, Brown, & Galbraith, 2010). It seems important to develop mathematical modelling competency and sub-competencies related to the sub-processes of the modelling cycle.

Mathematical modelling problems are more challenging than traditional word problems, or application problems. Word problems (common in school textbooks) usually follow prior instruction for a specific theme in mathematics and relate only to a segment of the real world (COMAP-SIAM, 2016). Some authors refer to word problems as Level 1 modelling problems (Tan & Ang, 2012). When solving such problems, the modelling process is limited to mathematisation, mathematical procedures following on prior instruction, and direct interpretation. An application problem could be compared to a Level 2 modelling problem (Tan & Ang, 2012) as context and meaning is added to the word problem and required in the solution.

These problems are common in assessment tasks at the secondary level. However, students still do not have an opportunity to put their analysis back into a real-world situation (COMAP-SIAM, 2016). Also, the attempt to contextualise the mathematics may make the mathematical work seem completely irrelevant. Mathematical modelling problems, or Level 3 modelling problems (Tan & Ang, 2012), are open ended and generated from a real-world situation that requires the complete modelling process. No clear

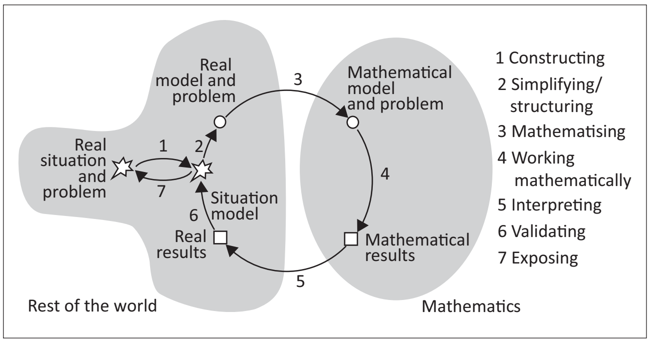

FIGURE 1: The modelling cycle according to Blum and Leiß (2007).

pathways are suggested in such problems and the modeller will seek for any mathematics to solve the problem and do research about the context and assumptions about the context. These problems might not be common in assessment tasks at the secondary level, but the benefit of incorporating such problems in the mathematics classroom has been established.

The following examples from the secondary environment, where students are learning how to write equations and draw graphs of linear functions given the slope and vertical intercept, can explain the difference between those three levels:

- Example of a Level 1 problem*:* Emil works at a retail store that pays R100 per week plus R2 for each item he sells. Last week he sold 85 items. How much did he earn last week?
- Example of a Level 2 problem: Emil works at a retail store that pays R100 per week plus R2 for each item he sells. Write a linear equation representing the relationship between Emil's weekly income and the number of items he sells.
- Example of a Level 3 problem: The holidays are approaching, and your best friend Karen would like to make some money to purchase gifts. She found one job that will pay R2 per hour above the minimum wage. Another job offers to pay half the minimum wage plus commission in the amount of R2 per item she sells. Which job is better? Help Karen to make the decision.

For various examples of modelling problems and an overview of the literature see Niss and Blum (2020).

The teaching and learning of mathematical modelling are difficult mainly due to the cognitive demand of modelling activities. Student teachers often have misconceptions about what mathematical modelling entails (Anhalt & Cortez, 2016), do not always understand the value of such activities, have mixed feelings about the topic and feel under-prepared to teach mathematical modelling (Blum & Borromeo Ferri, 2016; Ng, 2013).

An example of an exploratory study with a sample of mathematics educators on difficulties in teaching mathematical modelling in France and Spain is reported by Cabassut and Ferrando (2017). Results confirmed that most educators were positive about mathematical modelling, although some educators lack self-confidence. Most difficulties were experienced in relation to teaching the topic. These difficulties were specific to time (e.g. time to prepare for tasks, time on tasks), students' involvement, and resources. Borromeo Ferri (2018) highlights four key competencies that should be developed in teacher education in order for teachers to teach mathematical modelling effectively and appropriately. The competencies are: (1) theoretical competency for practical work, (2) task competency for instructional flexibility, (3) instructional competency for effective and quality lessons, and (4) diagnostic competency for assessment and grading. Developing these professional competencies in student teachers might narrow the gap between research and practice in teaching mathematical modelling. One way of narrowing the gap could be through design-based research (DBR) which is linked particularly well with the teaching and learning of mathematical modelling.

Characteristics of design-based research and design principles

DBR is a flexible methodology aiming at improving practices through iterative cycles of analysis, design, development, and implementation via interventions, and it focuses on the collaboration between researchers and practitioners with the intention to *extend* and *identify* new design possibilities (Abdallah, 2011). To identify key elements and to develop suitable design principles to guide the integration of mathematical modelling into the formal education of mathematics student teachers, Abdallah's view of DBR seemed suitable. Seven key characteristics of DBR are widely agreed upon (Anderson & Shattuck, 2012; Reeves & McKenney, 2013; Wang & Hannafin, 2004) and provided a suitable validation to structure the study: first, *DBR's authentic nature and naturalistic context* suggests a grounding in real-world contexts where participants can interact socially with one another, similar to everyday life. Second, *DBR generates design principles* that are contextually sensitive with the intention to improve practice. These principles are informed by theories, literature sources and the contextual variables, and are refined over multiple iterations. Third, *DBR's rigorous methodology* faces the difficulty of incorporating a variety of factors. Thus, the outcome should be the culmination of the interaction between designed interventions, human psychology, personal histories or experiences, and local contexts. To improve the design, data collection and analysis are conducted simultaneously, while credible findings and meaningful solutions to the envisaged problem are produced by the rigour and reflection of the DBR process. Fourth, DBR's *fragile, complex and 'messy' nature* requires mindful consideration from the researcher and a close collaboration between the researcher and participants to continuously refine the flexible design in the context. This close collaboration is possible through an appropriate well-structured or disciplined approach. Fifth, *DBR functions via unique processes* to transcend the local context, for example to refine the design continuously and iteratively (here the focus is on the processes). Sixth, *DBR transcends the local context* and attempts to provide an answer for why the problem occurs, what must be done and what we can learn from this to inform the practice of others (here the focus is on the elements related to the local context). Seventh, *DBR generates credible evidence and useful knowledge* that might also be used in another context. To ensure a purposeful and strategic intervention for student teachers to learn mathematical modelling, it is vital to consider these characteristics of DBR.

The product of a DBR study is to contribute to local (analogue to the situation and sample in this study) instructional theory through identifying key elements and fundamental principles.

Nieveen and Folmer (2013) emphasise that design principles, stemming from a DBR study, should provide insight into an educational intervention, and should communicate the purpose and context, key characteristics, design and procedural guidelines, as well as implementation conditions of the intervention. DBR has not often been used in studies to improve the professional development of teachers in mathematical modelling. However, one example of such a study to improve the didactical practices of primary school mathematics teachers through modelling was conducted by Biccard (2013). The study reported on in this article provided a unique opportunity to integrate mathematical modelling in the formal education of student teachers.

Methodology

By following a pragmatic approach (Creswell, 2013), considering the key characteristics of DBR, the researcher was looking for 'what works' in preparing student teachers for mathematical modelling activities.

Research design

With the suitability of DBR having been theoretically established earlier, the research was conducted via three main phases (view Figure 2). The *first* phase (cycles 1–3) was in preparation for the experiment with a focus on needs analysis. This phase was informed by an in-depth literature review, the researcher's personal experiences, ideas from other practitioners and specialists, and a pilot study. The *second* phase (cycles 4–6) focused on experimenting in the classroom keeping consistency and practicality in mind. This phase included all mathematical modelling activities structured over two iterations. The *third* phase (cycles 7 & 8) required a retrospective analysis of all qualitative and quantitative data with the intention to identify key elements and principles to prepare student teachers for mathematical modelling. Although the design of the three phases is supported by literature (see Reeves & McKenney, 2013), the phases reflected in Figure 2 included eight cycles, uniquely crafted for this study, that describe the design step by step.

The intention with the eight cycles was to distil contextually sensitive design principles that might be suitable in further studies or in other contexts. Both qualitative and quantitative data were collected in cycles 2, 4 and 6. The qualitative data provided information on mainly the shortcomings and progression in student teachers' mathematical modelling competencies and the quantitative data provided information on the change in student teachers' attitudes towards mathematical modelling activities based on this exposure.

A pilot study was conducted (see cycle 2 in Figure 2) one year before the implementation of the main intervention. The sample and conditions in the pilot study were similar to the main study.

On all occasions the context was a lecture room, in a real-life setting. The intention with the pilot study was to test the validity of the data collection instruments, to confirm the identified needs of participants and to improve the researcher's experience with a DBR strategy. Findings from the pilot study improved the preparation for the first iteration (cycle 4 in Figure 2), and some of its results have been published (see Durandt & Jacobs, 2014; Jacobs & Durandt, 2017). Similarly, findings from cycle 4 improved the preparation for the second iteration in cycle 6 and some results have been published (see Durandt & Lautenbach, 2020). Each data collection cycle (2, 4 & 6) was followed by a cycle where the results were conceptualised and draft design principles were identified and refined (cycles 3, 5 & 7). Through this cyclic process the findings from one cycle were incorporated in the design of the next cycle ensuring a continuous refinement of the key elements and principles relevant to the learning of mathematical modelling in this context.

The pilot study (cycle 2) consisted of one session of 90 minutes and participants acted as modellers. Iteration 1 (cycle 4) was similar to the pilot study. In iteration 2 (cycle 6) participants acted as both modellers and teachers of modelling. Four sessions of 90 minutes each were planned, two sessions allocated to each role. Rigour was maintained in

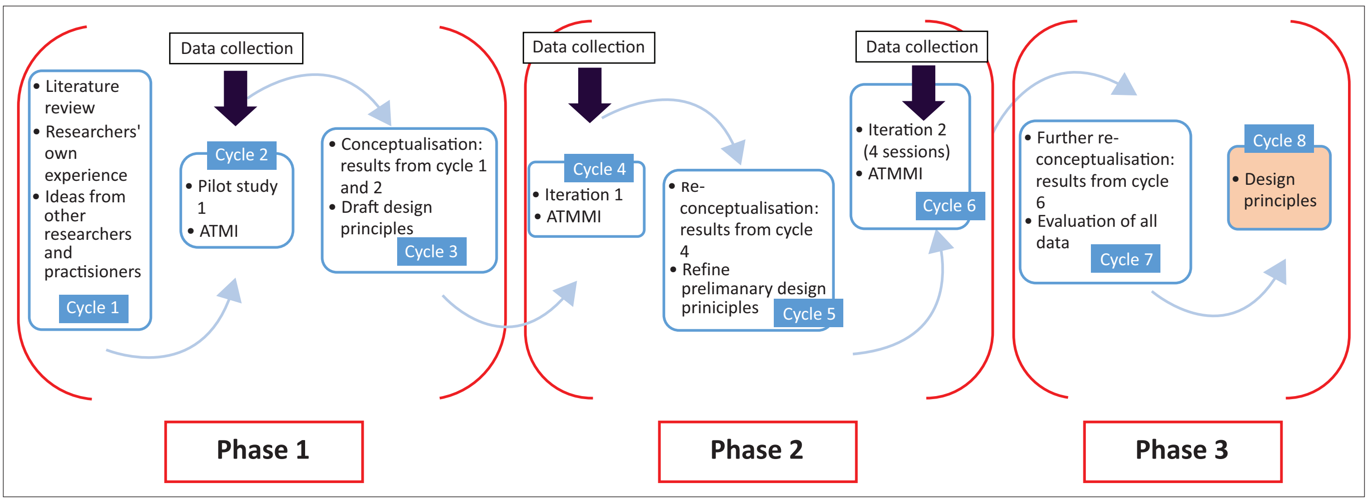

FIGURE 2: The cyclic process of design-based research (DBR) in this study structured over three phases involving eight cycles.

all research phases by following all activities as planned and a methodological specialist attended the sessions. Multiple data collection instruments were used throughout the research phases to identify key elements and fundamental principles relevant when student teachers learn mathematical modelling. These instruments are related to learn the cyclic process of mathematical modelling (worksheet and poster documents), to identify shortcomings and capture experiences (open-ended questionnaire), and to determine student attitudes towards mathematical modelling (ATMMI). For the focus of this article, only data collected from cycles 4 and 6 will be reported on, which mainly informed the retrospective analysis.

Iterations

In cycles 4 and 6 (see Figure 2) participants were exposed to modelling activities that ranged from solving easier tasks (like an application problem or Level 2 modelling example; see above for an explanation) to more challenging, open-ended and complex modelling tasks (like a Level 3 modelling example; see above for an explanation and below for an example). The intention with the range in complexity of tasks was to develop mathematical modelling competencies in student teachers, for them to learn sub-competencies and develop more positive attitudes towards such activities over time. The tasks asked participants to be modellers themselves, but also teachers of modelling. While acting as modellers themselves, they were expected to work their way through the modelling cycle in groups and record their work on a predesigned worksheet. These activities were included so that student teachers might develop both theoretical and task competencies as described by Borromeo Ferri (2018). One such example is discussed in a later section and for other examples of modelling activities and tasks used in the study, see Durandt (2018). In groups, they participated in discussions, designed their own modelling tasks, reflected on and evaluated their own and others' work, prepared posters and presented proposed solutions. In these activities, participants acted in both roles, as modellers themselves and also as teachers, with the intention to develop instructional and diagnostic competencies as emphasised by Borromeo Ferri.

Participants

A population of 55 third-year student mathematics teachers from a large public university in South Africa participated in the main study that was completed in 2018. The participants did not share their mathematics class with other mathematics students (like engineers or computer science students) and this separation was automatically done by the university system. The participants were arranged in the same 10 comparable groups in all iteration phases by using purposive sampling procedures. Relatively small groups of four to six members were formed, with each group containing at least a high, a moderate and a low achiever. The achievement level was determined by their formal course marks in mathematics. The group selection was mandatory (determined by the researcher) and purposeful in planning for multiple dependent variables such as the complexity of the modelling activities, student teachers' initial or limited exposure to mathematical modelling tasks, and the envisaged and expected valuable collaboration among participants.

Instruments to learn mathematical modelling

Student teachers were exposed to mathematical modelling tasks to learn modelling themselves, to develop mathematical modelling competencies and sub-competencies and with the intention to develop positive attitudes towards the topic. One example is the '*Location Problem*' (view Figure 3) where participants (in groups) were asked to use the data and make a recommendation to the Department of Town and Regional Planning on the best location for a day-care centre for toddlers.

Participants were expected to work through the processes of the modelling cycle, record their work on a predesigned worksheet with a four-step plan (view Table 1), similar to the idea of a plan used in other studies like the 'solution plan' used in the DISUM study (Schukajlow, Kolter, & Blum, 2015) and aligned with the main processes stemming from the modelling cycle (view Figure 1).

Real-world data on the 'Location Problem' generated on a section of the city's street network within a 1-hour period.

- Both First and Second Street are one-way streets from north to south.
- Third Avenue is a one-way street from west to east, but Fourth Avenue is a one-way street from east to west.
- Street corners are identified at Third Avenue and First Street, Third Avenue and Second Street, First Street and Fourth Avenue, Second Street and Fourth Avenue.
- 180 cars enter First Street.
- 70 cars enter Second Street.
- 200 cars leave First Street.
- 30 cars leave Second Street.
- 200 cars enter Third Avenue.
- 200 cars enter Fourth Avenue.
- 20 cars leave Third Avenue.
- 400 cars leave Fourth Avenue.

FIGURE 3: Real-world data from the '*Location Problem*' to inform a recommendation on the best location for a day-care centre for toddlers.

TABLE 1: Elements of the predesigned worksheet: Four-step plan.

Elements	Activities expected from the groups
Step 1: Mathematisation	Understand the problem. Structure the information. Make assumptions to simplify the problem. Represent the problem in mathematical form.
Step 2: Working within mathematics	Solve the problem mathematically, use several mathematical methods and tools, use resources like textbooks or information and communication technologies.
Step 3: Interpretation	Interpret the mathematical solution within the context of the real-life situation.
Step 4: Reflection	Review the assumptions and the limitations of the mathematical model and the solution to the problem. Review the mathematical methods and tools used and make suggestions for alternative solutions. Provide a suitable answer to the original question and support the answer with valid reasons.

After solving the task, all groups participated in discussions, both in groups and as a whole class. Then, each group prepared and presented a poster to demonstrate visually how they used the modelling cycle to find a possible solution to the problem. See Figure 4 for an example from group 9 after solving the '*Location Problem*' in iteration 2.

They also evaluated their own work and the work of others by using an evaluation sheet (selecting one of three criteria: high, medium, low). Later, at the end of each iteration, participants reported their experiences by answering an open-ended questionnaire individually.

The questionnaire included a section on biographical information of participants as well as a section on their perceptions of the mathematical modelling experience and support they might require. One example of a question from the questionnaire is: How can the university further support you (during your teacher training) in becoming an even more effective teacher of mathematical modelling (make concrete suggestions)? The qualitative data collection instruments (worksheets, poster and open-ended questionnaire) were designed at the end of cycles 1, 3 and 5.

Content analysis methods were used to analyse the data (Saldaña, 2016). Students' worksheet and poster documents were marked according to a framework deduced from the sub-processes of the modelling cycle (see Figure 1) and compared through the cycles (see Table 2). Data collected through the open-ended questionnaire were analysed according to themes, and further separated in categories and sub-categories (see Figure 5). This was done via pen and paper and using Atlas.ti software. Strategies to maintain the trustworthiness of the qualitative component of this study included a thick description of the methodology (in section 3) and an external subject specialist confirming coding categories and qualitative findings (as suggested by Creswell, 2013).

FIGURE 4: Poster example from group 9 after solving the '*Location Problem*' in iteration 2 and showing how they moved through the processes in the modelling cycle.

Instrument to determine attitudes

The Attitudes Towards Mathematical Modelling Inventory (ATMMI) was used to gain information individually regarding student teachers' attitudes towards mathematical modelling at the end of each iteration.

The ATMMI, adapted from Schackow (2005) that followed the original ATMI instrument from Tapia and Marsh (2004), is a locally tested instrument that consists of 40 Likert-scale items arranged from 'strongly disagree' to 'strongly agree' (5 possible responses). The items were grouped in four sub-scales: *enjoyment* (10 items to determine whether mathematical problem-solving and modelling challenges were considered enjoyable for participants), *value* (10 items, to determine whether mathematical modelling knowledge and skills were considered worthwhile and necessary for participants), *self-confidence* (15 items, to determine the expectations about doing well in respect of mathematical modelling, and how easily modelling was mastered by participants) and *motivation* (5 items, to determine the desire of participants to learn more about mathematical modelling and to teach the topic). Data were collected on paper at the end of iteration 1 (cycle 4) and at the end of iteration 2 (cycle 6). The Statistical Software Package for the Social Sciences (SPSS, version 24) was used to analyse the data. Internal consistency was confirmed by acceptable Cronbach's alpha coefficients (in all sub-scales > 0.8). The results were similar to previous recorded alpha values by Schackow and Tapia and Marsh. Content validity was confirmed by mathematical modelling specialists from other South African universities, construct validity by previously recorded factor analysis (see Tapia & Marsh, 2004) and sight validity through the pilot study. A non-parametric test (Wilcoxon Signed Ranks test) was used to evaluate the change in attitudes of student teachers towards mathematical modelling after exposure to a series of mathematical modelling activities, and a parametric test (one-sample t-test) was used to

TABLE 2: Overall mean scores as percentages related to developing competencies (Theme 1) collected in cycles 4 and 6 via worksheets and poster presentations.

Variable	Data cycle 4	Data cycle 6			
	Worksheet MM task %	Session 1 worksheet TP task %	Session 2 worksheet MM task %	Session 3 poster & presentation %	Session 4 poster & presentation %
Modelling sub-competency:					
Modelling cycle	68	100	78	65	70
Real-world interpretation	65	n/a	90	50	70
Mathematical sub-competency:					
Mathematical representation	80	90	100	65	90
Content selection & calculations	50	73	85	60	70
Working with mathematical models:					
Model efficiency	40	86	82	n/a	n/a
Model applicability	60	n/a	93	n/a	n/a
Teacher competencies:					
Preparation work	n/a	n/a	n/a	n/a	90
Facilitation work	n/a	n/a	n/a	90	n/a

n/a, no data collected during this cycle or session regarding this aspect; MM, mathematical modelling; TP, traditional problem-solving.

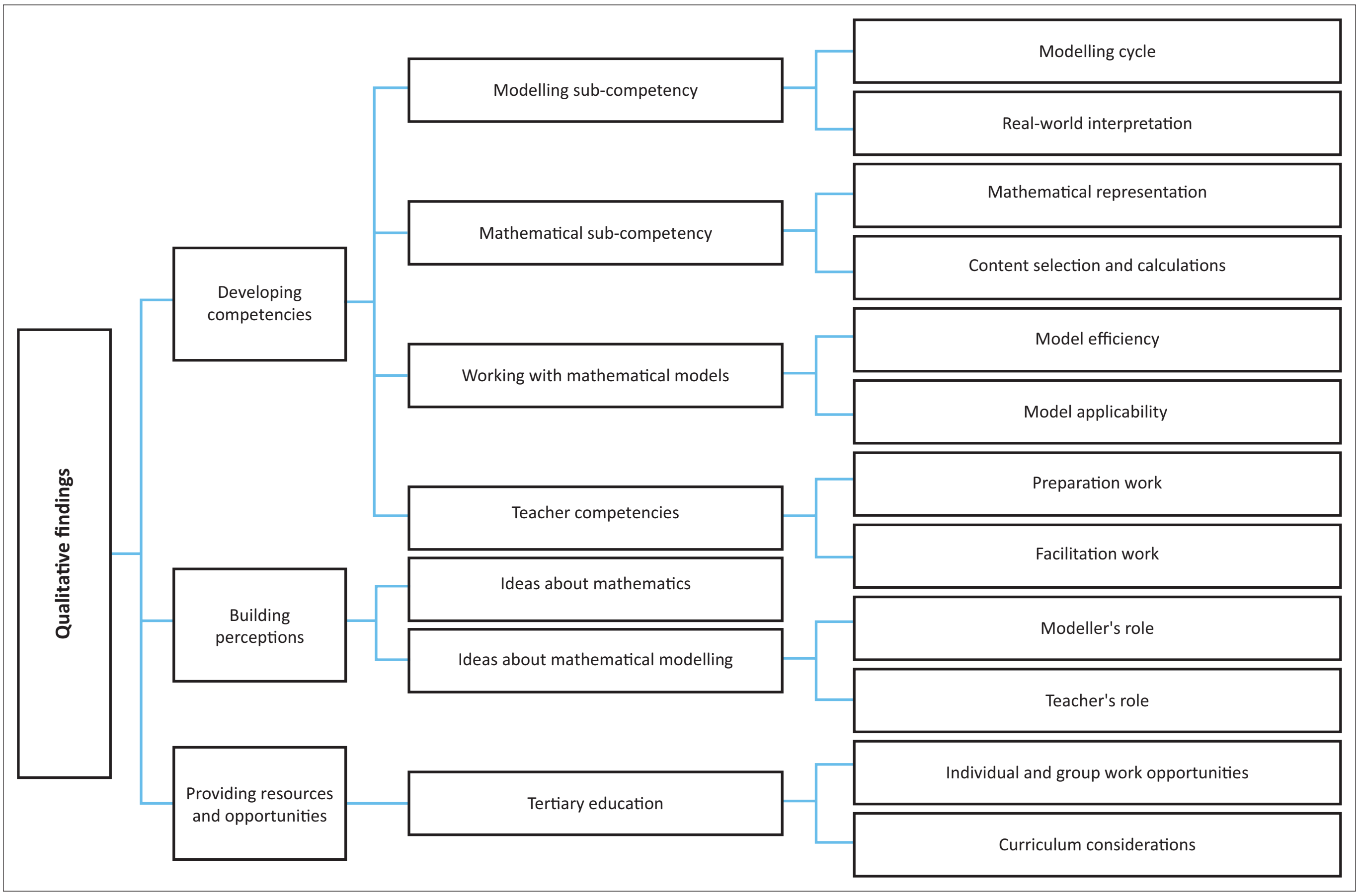

FIGURE 5: Reflective analysis and evaluation of all qualitative data: themes, categories, and sub-categories.

determine the effectiveness of the methodological design. Due to the sample size (30+) and the robustness of statistical approaches (Pallant, 2010), the parametric test was a possibility.

Ethical considerations

Standard ethical measures were taken according to the literature (Creswell, 2013; Teddlie & Tashakkori, 2009) and the procedures at the university (ethics clearance number 2015-024). At the beginning of the modelling activities in cycles 2, 4 and 6, participants were briefly informed about the planned activities. Participation was voluntary and participants signed an informed consent form. The researcher had a dual role for the purpose of this DBR study, both designer and researcher. This is a limitation and was purposefully addressed by balancing the roles, and through the observations recorded by the methodological specialist during implementation.

Findings and results

Student teachers' shortcomings and progression in modelling activities

Based on data collected from the worksheets, posters and open-ended questions (collected in cycles 4 & 6, view Figure 2) themes were identified reflecting student teachers' shortcomings and progression in mathematical modelling activities.

All themes focused on elements and principles related to *developing competencies* (Theme 1), *building perceptions* (Theme 2) and *providing resources and opportunities* (Theme 3). The categories and sub-categories included in these themes intend to sketch a picture of the challenges participants experienced throughout the intervention. Figure 5 shows an overview of the themes, with categories and sub-categories.

The first theme, *developing competencies*, groups the needs (to a lesser or greater extent) of student teachers to develop particular competencies as they proceeded through the intervention in this study. There were four sub-categories: modelling competencies, mathematical competencies, utilising mathematical models and facilitator competencies (see Table 2). Modelling competencies refer to how participants proceeded through the different phases of the modelling cycle and interpreted their findings in terms of real-world relevance. Mathematical competencies refer to the mathematical representation of real-world problems, relevant mathematical content selection and the need for relatively accurate calculations. Utilising mathematical models refers to an investigation of model efficiency (by criteria from Meyer, 2012) and an exposure to real-life model applicability. Teacher competencies describe the preparation work (e.g. designing of a modelling task) and the facilitation work (e.g. facilitating class discussions reflecting on the modelling process) of participants as they took on the role as teachers of modelling. For example, in the '*Location Problem*' (see sub-section 3.4) as participants took on the role of modellers themselves, they structured the information and represented the one-way streets and number of cars travelling along specific roads visually (like in the display in Figure 4), but they had trouble constructing a suitable mathematical model (a linear system of four equations with four variables) to represent the situation. Even if they managed to construct a suitable mathematical model, some groups struggled to solve the system of equations. Such difficulties caused blockages in the modelling cycle. One group made a calculation error (where the variables that represent the number of cars travelling along a specific road were equal to negative values) and they struggled to interpret their mathematical result within the context of the example. Table 2 represents the overall mean scores as percentages of participants in groups who mastered the specific competency.

The second theme, *building perceptions*, highlights the importance (based on former and current opinions) of developing a positive disposition in student teachers towards mathematical modelling activities (for example the '*Location Problem*'). This theme is divided into two sub-categories: ideas about mathematics and ideas about mathematical modelling (informed by data findings from the open-ended questionnaire). Ideas about mathematics refer to participants' thinking about mathematics as a subject, the teaching thereof and valuing the subject. Ideas about mathematical modelling are related to participants' disposition based on their exposure to modelling activities as modellers themselves and as facilitators of such activities throughout their formal education (for example, in this study their exposure to modelling activities in cycles 4 and 6; see Figure 2). Qualitative data collected from the open-ended questionnaire (in cycles 4 & 6) showed:

- Approximately half of the participants commented on a positive and enjoyable school learning experience in mathematics. They used phrases like '*interesting and a good challenge*', '*my favourite subject*', and '*key to success*'.
- A fifth of the participants described their school learning in mathematics in a traditional and partially negative way. They experienced the class as uninteresting and could not understand the link between mathematics and real life. One participant commented '*Mathematics was theoretically based and failed to integrate it into a real-life problem*', while another wrote '*bored in class*' and '*confusing*'.
- Two-thirds of the participants commented positively on their involvement in the modelling activity. Some of the participants found the experience extremely interesting, exciting and stimulating, and realised the meaningfulness of mathematics. For others, it began with some frustration, but it improved, and they were willing to learn. For example, participants commented '*The topic is scary when still reading but once you get the way of using it, it is very interesting*'.
- The majority of participants (41) described the modelling task as overwhelming and challenging, while a quarter of the participants felt that they had some idea to solve the problem. Participants mentioned key aspects as challenges in the modelling cycle, such as finding a strategy and deciding on a point of departure, selecting relevant information and expressing the information mathematically. Comments such as '*We were confused on where to start and what to start with*', and '*we could not understand the problem*' explained their point of view. They also mentioned the need to think 'out of the box'.
- Findings from the data revealed 49 participants indicated the modelling task was sufficiently real-world related and realistic.

The third theme, *providing resources and opportunities*, indicates the required support highlighted by student teachers to develop in the teaching and learning of mathematical modelling throughout their professional education. This theme is divided into three sub-categories (informed by data findings from the open-ended questionnaire): pre-tertiary education, tertiary education and post-tertiary education. The categories pre-tertiary education and post-tertiary education are beyond the scope of this study although the researcher recognised from the findings that support during these phases could contribute towards the development of student teachers' competencies in mathematical modelling and the further enhancement thereof. The category tertiary education refers first to the group work opportunities required by student teachers to gain modelling competencies (as modellers and facilitators) and to develop proficiency in collaborative actions, and second to particular curriculum considerations to ensure continuous contextualised exposure (to content and teaching methodologies) of mathematical modelling activities. For example, participants worked together in small groups to find a real-life solution for the '*Location Problem*'.

Qualitative data collected from the open-ended questionnaire (in cycles 4 & 6) showed:

- Approximately half of the participants commented positively on the collaboration and highlighted the manner in which the group worked as a team, the manner in which the modelling problem stimulated interaction, and the manner in which all group members shared ideas. For example, a participant commented '*Our group worked well together and it seemed that we liked the challenge*'. Contrary to this, others experienced the group work as being rather negative. They mentioned challenges such as language and cultural barriers, group members' contributions (or lack thereof) and no agreement on a strategy, inactive group members, confusion and, in some cases, group members being just concerned with getting an answer.
- Approximately two-thirds of the participants (32) preferred a mandatory group allocation, while one-third (19) preferred a voluntary selection.
- Most participants (48) confirmed their groups' modelling abilities improved over the period of exposure to the modelling activities.

- The majority of participants (51) regarded themselves as semi-active or active (4 non-active) in their particular group. Similarly, they (50) also viewed themselves as constructive contributors. They gave a number of reasons for their participation, such as the group member who came up with the solution, or completed the worksheets, presented and took on the role as group leader.
- A majority (54) indicated they would benefit from continuous exposure to material regarding the teaching of mathematical modelling and the approach to modelling tasks. For example, a participant wrote '*I would benefit a lot for it will give me skills and knowledge*'. Then again, 46 participants (with 5 no responds) confirmed they would participate in discussions regarding the teaching and learning of mathematical modelling. Most respondents approved the use of an electronic platform (such as social media or an electronic learning environment), but some preferred face-to-face contact.

Reflective analysis of quantitative data

Within each sub-scale, the researcher compared total scores and investigated cross-tabulation results for respective items. Due to the comparative nature of the analyses in DBR phase 3, only cases displaying the necessary information in both iterations were considered (enjoyment 44 cases, value 43 cases, self-confidence 44 cases, and motivation 43 cases). Hence, missing data were excluded pairwise. The Wilcoxon Signed Ranks test as statistical technique is regarded as suitable to answer the research question related to change in attitudes over time (Pallant, 2010). This test converts scores to negative or positive ranks and compares them at iteration 1 and 2.

Enjoyment: More positive ranks (26) than negative ranks (14) on the *enjoyment* scores of participants were detected, but no significant change in *enjoyment* mean scores following participation in the mathematical modelling intervention over two iterations. Table 3 shows $z = -1.86$ and $p = 0.06$, with a small to medium effect size ($r = 0.20$) according to the criteria by Cohen (1988).

Findings from cross-tabulation data in the *enjoyment* sub-scale revealed at the end of iteration 2 that most participants (over 80%) mildly to strongly agreed on the following aspects: (1) mathematical modelling is a very worthwhile topic and they wanted to further develop their mathematical modelling skills, (2) the process taught them to think and they recognised the importance of the topic in everyday life, (3) they were not sure how the topic can be utilised in learning mathematics although it seems important for mathematics students for any grades of teaching, (4) the usefulness of studying mathematical modelling at a higher education level, and (5) their belief that the topic will support them with problem-solving in other areas and a strong background in modelling could help any mathematics teacher.

TABLE 3: Test statistics for the enjoyment sub-scale.

Test statistics†	Enjoyment (Iteration 2) – Enjoyment (Iteration 1)
Z	-1.856‡
Asymptotic significance (2-tailed)	0.063

†, Wilcoxon Signed Ranks test; ‡, Based on negative ranks.

Value: More negative ranks (24) than positive ranks (17) on the *value* scores of participants were detected and no significant change in *value* mean scores following participation in the mathematical modelling intervention. Table 4 shows $z = -0.182$ and $p = 0.86$, with a small to medium effect size ($r = 0.20$) according to the criteria by Cohen (1988).

Findings from cross-tabulation data in the *value* sub-scale revealed at the end of iteration 2 that participants mildly to strongly agreed that they: (1) got a great deal of satisfaction out of solving a mathematical modelling problem (70.2%), (2) enjoyed being involved in a mathematical modelling session (66.7%), (3) liked to solve real-world problems in mathematics (69.8%), (4) preferred a mathematical modelling task rather than writing an essay (73.2%), (5) thought they liked the topic (59.5%), (6) preferred real-world problems to other mathematical themes (29.3%), (7) found the topic very interesting (76.2%), (8) were comfortable expressing their own ideas on how to solve a modelling problem (69.8%), and (9) were comfortable suggesting possible solutions to a modelling problem (73.1%). Furthermore, participants (86%) mildly to strongly disagreed with the statement that mathematical modelling is dull.

Self-confidence: More positive ranks (26) than negative ranks (15) on the *self-confidence* scores of participants were detected and no significant change in *self-confidence* mean scores following participation in the mathematical modelling intervention. Table 5 shows $z = -1.42$ and $p = 0.16$, with a small effect size ($r = 0.15$) according to the criteria by Cohen (1988).

Findings from cross-tabulation data in the *self-confidence* sub-scale at the end of iteration 2 revealed that participants mildly to strongly disagreed with the following: (1) mathematical modelling is a feared topic (50%), (2) it created a feeling of dislike (69.8%), (3) their minds went blank when confronted with the topic (62.8%), (4) mathematical modelling made them nervous (51.2%), and uncomfortable (67.5%), (5) they experienced terrible strain in a mathematical modelling session (66.7%) and just thinking about the topic made them nervous (67.4%), (6) they were confused in a modelling session (55.8%),

TABLE 4: Test statistics for the value sub-scale.

Test statistics†	Value (Iteration 2) – Value (Iteration 1)
Z	-0.182‡
Asymptotic significance (2-tailed)	0.856

†, Wilcoxon Signed Ranks test; ‡, Based on negative ranks.

TABLE 5: Test statistics for the self-confidence sub-scale.

Test statistics†	Self-confidence (Iteration 2) – Self-confidence (Iteration 1)
Z	-1.420‡
Asymptotic significance (2-tailed)	0.156

†, Wilcoxon Signed Ranks test; ‡, Based on negative ranks.

and (7) they felt insecure when attempting mathematical modelling (64.3%). However, participants mildly to strongly agreed with the statements: (1) mathematical modelling did not scare them (45.3%), (2) they had self-confidence with the topic (41.9%), (3) they would be able to solve such a problem without too much difficulty (26.6%), (4) their expectation would be to do fairly well in future modelling sessions (64.3%), (5) they will learn mathematical modelling easily (45.2%), and (6) they believed they were good at solving real-world problems (48.9%).

d) Motivation: More positive ranks (25) than negative ranks (14) on the *motivation* scores of participants were detected and no significant change in *motivation* mean scores following participation in the mathematical modelling intervention. Table 6 shows $z = -1.71$ and $p = 0.09$, with a small to medium effect size ($r = 0.20$) according to the criteria by Cohen (1988).

Findings from cross-tabulation data in the *motivation* sub-scale revealed at the end of iteration 2 that participants mildly to strongly agreed with: (1) they could pass a course on mathematical modelling for mathematics teachers (62.8%), (2) they were willing to learn more about mathematical modelling in future (65.1%), (3) the challenge of mathematical modelling is appealing (54.7%), and (4) they would be keen to enrol for a course on mathematical modelling for mathematics teachers (65.2%). Furthermore, participants mildly to strongly disagreed that they would like to avoid teaching mathematical modelling (67.5%).

Finally, the researcher investigated the percentage change in scores between iteration 1 and 2 for all sub-scales (by utilising the formula:

$$\frac{Iteration\ 2\ scores - Iteration\ 1\ scores}{Iteration\ 1\ scores} \times 100). \qquad [Eqn1]$$

The purpose of this test was to evaluate the effectiveness of the mathematical modelling intervention over two iterations on participants' attitudinal scores and more broadly on the methodological design. Table 7 shows positive percentage changes in all sub-scales, and significant percentage changes in both the *enjoyment* ($t[43] = 2.12$, $p < 0.05$ two-tailed) and *motivation* sub-scales ($t[42] = 2.16$, $p < 0.05$ two-tailed).

Discussion

At the beginning of the study, student teachers had misconceived ideas of the modelling process according to the data collected in cycle 4, and they struggled to find their way through the modelling cycle. They experienced challenges in each phase of the modelling cycle, such as mathematisation, working with mathematics, interpretation and reflection. These findings correlated with findings from other studies (compare Zeytun, Cetinkaya, & Erbas, 2017). Towards the end of the study, student teachers displayed an improved understanding of the modelling process which was evident in their poster presentations in cycle 6. Anhalt and Cortez (2016) presented similar results. Thus, in order to design an intervention for student mathematics teachers to learn mathematical modelling during their formal education in the South African context, the designer is best advised to emphasise the development of student teachers' mathematical and modelling competencies, capabilities in utilising mathematical models and facilitator competencies, all of which should focus on demonstrating practicality. The following procedures, which flow from the qualitative findings after student teachers were exposed to a series of modelling activities, might be considered: (1) providing an *authentic modelling activity* with a familiar mathematical content and real-world context, also reported by Ikeda (2013), (2) providing opportunities in demonstrating the *applicability of mathematical models* in a real-world context (compare Meyer, 2012), (3) promoting *active participation* and *socially constructed knowledge* by means of class discussions and collaboration (supported by Anhalt & Cortez, 2016), (4) encouraging *individual development* and *building of self-confidence* in mathematical and modelling competencies (also supported by the ATMMI results; see sub-section 4.2), (5) providing *support by scaffolding* the processes in the mathematical modelling cycle (supported by other studies, for example Blum 2015), and (6) creating an opportunity to practise aspects of *teaching of modelling*, develop competency in preparation work (e.g. the selection of mathematical modelling tasks), and facilitation work.

Furthermore, student teachers were able to build an appropriate belief about mathematical modelling and to develop a positive disposition towards it by participating in the series of modelling activities. At the beginning of the intervention, they started with a vague idea and even misconceptions about mathematical modelling, but towards the end, most participants had developed an understanding of the modelling process. They also had predetermined ideas about mathematics (compare Jacobs & Durandt, 2017). This correlated with beliefs about school mathematics explained in Ärlebäck (2009) and the way students act in a typical

TABLE 6: Test statistics for the motivation sub-scale.

Test statistics†	Motivation (Iteration 2) – Motivation (Iteration 1)
Z	-1.708‡
Asymptotic significance (2-tailed)	0.088

†, Wilcoxon Signed Ranks test; ‡, Based on negative ranks.

TABLE 7: One-sample *t*-test for percentage changes in each sub-scale.

Variable	Test value = 0					
	T	*Df*	Significance (2-tailed)	Mean difference	95% confidence interval of the difference	
					Lower	Upper
Percentage change in *Enjoyment* scores	2.118	43	0.040	3.15057	0.1511	6.1500
Percentage change in *Value* scores	1.247	42	0.219	4.34311	-2.6836	11.3698
Percentage change in *Self-confidence* scores	1.584	42	0.121	6.48054	-1.7779	14.7390
Percentage change in *Motivation* scores	2.156	42	0.037	13.34177	0.8560	25.8275

manner based on their mathematics belief system (Kaiser, 2017). They therefore required opportunities to build positive perceptions about mathematical modelling which is also evident from the ATMMI findings.

In the final ATMMI results, a significant positive percentage change in student teachers' *enjoyment* scores over the two iterations was indicated. Positive enjoyment scores in mathematical modelling can be linked to positive feelings of satisfaction, a willingness to participate in activities, and the persistence to develop competencies. Özdemir and Üzel (2012) also reported positive enjoyment results. Likewise, a significant positive percentage change in student teachers' *motivation* scores over the two iterations was indicated in the ATMMI results. An increase in motivation could encourage student teachers to become interested in mathematical modelling and to display the curiosity needed to continue studying mathematical modelling. Thus, student teachers could grow in confidence when participating in modelling activities. These results contradict some findings from Kreckler (2017) where students at the secondary level were exposed to a series of modelling activities and although a significant increase in the global modelling competence independent of grade and topic was reported no significant changes in motivation could be identified. The ATMMI showed a positive although not significant percentage change in student teachers' *self-confidence* scores over the two iterations. Higher confidence equals lower anxiety and higher confidence scores relate to performance and ability and provoke enjoyment (compare Maxwell, 2001). The ATMMI *value* scores also reflected a positive but not significant percentage change, although at the end of the intervention, more than half of the student teachers could not really understand the value of mathematical modelling. The value of learning mathematical modelling is related to personal goals, and the person's perceived usefulness of mathematical modelling. One reason might be that the specific modelling tasks used in the intervention did not link to the personal preferences and realities of some participants and, as a result, they could not see the 'gain' in these activities. Özdemir, Üzel and Özsoy (2017) explained value is determined by teachers' knowledge, thus a lack thereof could also be regarded as a reason why student teachers did not fully value the modelling activities in this study.

Overall, the increased scores indicated a positive change in aspects (enjoyment, self-confidence, value and motivation) determining the attitudes of student teachers towards mathematical modelling, and ultimately creating a more positive disposition towards the topic. Thus, in order to design an intervention for student mathematics teachers to learn modelling during their formal education in the South African context, the designer is best advised to emphasise the building of student teachers' perceptions about mathematics and mathematical modelling, to emphasise practicality via some procedures like: (1) establishing an *inviting modelling climate* to promote mathematical modelling practices among student teachers, (2) creating an *authentic context*, to which student teachers could relate, and that would reflect the real-world usability of the knowledge, and (3) promoting *reflective activities,* such as class discussions to replicate the authentic context, and to demonstrate the value of real-world application.

Student teachers required multiple opportunities to learn mathematical modelling over a period of time. Available resources, like knowledge, and tools such as technology, as well as practical matters, such as time constraints, were other concerns. Opportunities to learn mathematical modelling should involve activities with student teachers both as modellers themselves, and as facilitators of modelling. From the beginning, student teachers were grouped in mandatory groups, and they experienced a number of challenges in understanding the mathematical modelling tasks, continuous participation of all group members and collaboration among members (also indicated by answering the question in the questionnaire). Thus, in order to design an intervention for student mathematics teachers to learn modelling during their formal education in the South African context, the designer is best advised to provide student teachers' with opportunities and required resources (both characteristics to demonstrate practical emphasis), via some procedures like: (1) promoting *continuous and frequent exposure* to mathematical modelling activities throughout student teachers' professional development, (2) grouping student teachers in *comparable groups* and rotating responsibilities among group members in order to ensure productive participation and the development of self-confidence in all members (compare Goos, 2004), and (3) managing the classroom regarding *practical matters* (e.g. time management and logistical arrangements).

The DBR approach, chosen for this study with a pragmatic view, resulted in rich data shedding light on student teachers' shortcomings and progression in modelling competency and their positive attitudinal experiences, similar to the results of other studies (e.g. Anhalt & Cortez, 2016; Biccard, 2013). Both the approach and the findings of this study could support the international discussion. A limitation is that DBR guides theory development but usually takes place through several iterations and this study could thus be further developed through more iterations. Additionally, a narrower lens on aspects of mathematical modelling (e.g. reflective activities) could enhance metacognitive development on an individual level.

Conclusion

In this study, a cohort of third-year mathematics student teachers at a South African institute of higher education (grouped in 10 groups) were exposed to a series of well-planned mathematical modelling activities over two iterations – both as mathematical modellers themselves and as teachers of modelling. Both qualitative and quantitative data were collected during the DBR phases. Qualitative data were analysed by means of content analysis methods, and quantitative data were analysed with SPSS. Both the *t*-test (parametric) and the Wilcoxon Signed Rank test (non-parametric alternative) were utilised to determine participants' mean change in attitudes towards mathematical modelling (concerning motivation, value, self-confidence, and enjoyment) over time. Finally, a reflective analysis on all data contributed to desirable elements and underlying principles that might inform local instructional theory. The elements and principles ought to develop student teachers' mathematical modelling competencies, building positive perceptions, and to provide them with resources and opportunities to learn modelling themselves and to learn how to teach modelling. Towards the end of the study, student teachers displayed an improved understanding of the modelling processes and sub-processes and showed significant positive percentage changes in their enjoyment and motivation towards mathematical modelling over the course of the study.

Awareness of the student mathematics teachers' experiences when mathematical modelling is learnt through incorporating a selection of key elements and fundamental principles in their formal education could contribute to decreasing the cognitive burden of student teachers and cultivating positive experiences.

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Competing interests

The author has declared that no competing interest exists.

Authors' contributions

I declare that I am the sole author of this article.

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Data availability

The data that support the findings of this study are available on request from the author. The data are not publicly available due to restrictions (i.e. information that could compromise the privacy of research participants).

Disclaimer

The views and opinions expressed in this article are those of the author and do not necessarily reflect the official policy or position of any affiliated agency of the author.

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