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Novice and expert Grade 9 teachers' responses to unexpected learner offers in the teaching of algebra

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In South Africa, limited studies have been conducted investigating responsive teaching and little is known about how teachers respond to unexpected events 'in the moment' that did not form part of their planning. In this article, we report how a Grade 9 novice and expert teacher responded to unexpected learner offers during the teaching of algebra using a qualitative case study approach. Three consecutive lessons for each teacher were video recorded, transcribed and analysed. Our units of analysis for episodes were teachers' responses to unexpected learner offers and we coded the responses as 'appropriate' or 'inappropriate'. Indicators used to highlight the degree of quality of the response were 'minimum', 'middle' and 'maximum' if a response was coded as appropriate to a learner's offer. Once lessons were analysed, the first author conducted video-stimulated recall interviews with each participant to gain insight into the two teachers' thoughts and decision-making when responding to unexpected learner offers. The findings from this study illustrated that the novice teacher failed to press learners when their thinking was unclear, chose to ignore or provided an incorrect answer when faced with an unexpected learner offer. Conversely, the expert teacher continuously interrogated learner offers by pressing if a learner offer was unclear or if she wanted learners to explain their thinking. This suggests that the expert teacher's responses were highly supportive of emergent mathematics learning in the collective classroom space.

Keywords: Responsive teaching; unexpected events; contingency; secondary school; algebra; novice and expert teachers

Introduction and background to the study

International literature on responsive teaching suggests that quality teaching is highly improvisational as teachers ask learners questions based on learners' thinking 'in the moment' (Sawyer, 2004). Moreover, teachers need to listen interpretively and respond constructively to learner offers rather than ignoring or dismissing a learner's offer (Davis, 1997; Jacobs & Empson, 2016). Borko and Livingston (1989) have noted that responding to unexpected learner ideas is an important albeit challenging facet of effective teaching.

Abdulhamid and Venkat (2018) have acknowledged that limited research has been conducted in the South African terrain concerning responsive teaching and the research base is at a distance from responsive teaching described in the international literature. Evidence of responsive teaching at primary school level in South Africa suggests that 'in the moment' responsiveness is rare due to lessons being highly scripted and that there is a lack of evaluation of learners' offers accompanied by chorusing in the collective classroom space (Abdulhamid & Venkat, 2018; Hoadley, 2012). However, the situation at secondary level is different since empirical evidence suggests that teachers often evaluate learners' offers using an Initiation-Response-Evaluation (IRE) format such as a question-and-answer exchange but the nature of their engagement with learner offers is often superficial (Brodie, 2007). Currently, little is understood about how teachers respond to learners when faced with unexpected incidents that did not form part of their planning. This knowledge in action demonstrated when teachers deal with unexpected events forms part of the contingency dimension associated with the Knowledge Quartet formulated by Rowland, Huckstep and Thwaites (2005). In recent years, Rowland, Thwaites and Jared (2015) have proposed that there are three triggers of contingency associated with responding to unexpected events and these include (1) responding to unexpected learner offers, (2) teacher insight and (3) the sudden (un)availability of pedagogical tools such as technology during teaching. Based on the paucity of research in South Africa associated with contingency, we were interested to explore how Senior Phase expert and novice teachers (specifically at Grade 9) responded to unexpected learner offers while teaching a fundamental topic such as algebra.

Our focus on algebra[1] emanates from difficulties noted by examiners of high stakes examinations such as the matric examination: the Department of Basic Education (DBE) notes the 'algebraic skills of the candidates are poor' and that the majority of learners 'lacked fundamental and basic mathematical competencies which could have been acquired in the lower grades' (DBE, 2019, p. 179) such as Grade 8 and Grade 9. In both international and local terrains, algebra has been continuously noted as being abstract for learners and the transitioning from arithmetic to algebra is difficult since learners erroneously link arithmetic principles with algebra (Herscovics & Linchevski, 1994; MacGregor & Stacey, 1997). In both South Africa and internationally, attention has been predominantly focused on learner errors since errors and misconceptions are believed to provide a lens for understanding learners' thinking (Gardee & Brodie, 2015; Makonye & Luneta, 2014; Pournara, Hodgen, Sanders, & Adler, 2016). However, Tsui (2003) notes that it is imperative that researchers in different domains such as language and mathematics direct their efforts to understanding how the knowledge bases of novice and expert teachers differ and explore how these knowledge bases are developed. The purpose of this study was to investigate how a novice and expert Grade 9 mathematics teacher responded to unexpected learner offers within their lessons and gain insight into their thoughts and decision-making when responding to learners via video-stimulated recall (VSR) interview.

The following two research questions guided the investigation:

1. How do novice and expert Senior Phase teachers respond to unexpected events in the teaching of algebra?
2. What are the novice and expert Senior Phase teachers' thoughts and decision-making in response to unexpected events during the teaching of algebra?

We begin this article with an overview of responsive teaching, and how it is seen in the South African context. We then provide a discussion on contingency in mathematics teaching, as a vehicle for responsive teaching. A discussion of why a focus on novice and expert teachers is presented. The literature base on algebra acted as a vantage point for understanding and commenting on the nature of teachers' responses to contingent moments. Details of the observed lessons are presented and then analysed based on a modified form of Weston's (2013) coding protocol. We present an interview outline involving questions that interrogate teachers' decision-making in response to contingent moments in the teaching of algebra. Subsequently, analysis and findings from the study are presented.

Literature review

Overview of responsive teaching

In the context of classroom discourse, responsive teaching refers to when a teacher follows up on learners' questions or offers which open up possibilities for mathematical learning in the collective classroom space. Over the last 40 years, IRE interactions have been widely studied by researchers and there are two main bodies of literature associated with responsive teaching: 'deficit' approaches and 'affordance' approaches (Abdulhamid & Venkat, 2018; Brodie, 2007; Mehan, 1979; Sinclair & Coulthard, 1975). The 'deficit' approach is associated with teachers listening evaluatively to learner offers, asking questions that are predetermined and not contingent on learner offers (Davis, 1997). Moreover, 'funnelling' is a common phenomenon associated with 'deficit' approaches wherein teachers narrow the questions to a point that learners can answer them with minimal cognitive effort (Bauersfeld, 1980; Stein, Grover, & Henningsen, 1996). Conversely, 'affordance' approaches refer to teachers asking 'authentic' questions that are not pre-planned and teachers let learner offers steer the trajectory of the lesson (Borko & Livingston, 1989; Jacobs & Empson, 2016; Nystrand, Gamoran, Kachur, & Prendergast, 1997; Sawyer, 2004). The improvisational and spontaneous nature of responsive teaching associated with 'affordance' approaches is a concerted effort between the teacher and learners (Sawyer, 2004). This is comparable to theatre actors working without a script as part of improvisation since the conversation taking place is unplanned and dependent on how interactions unfold between the actors (Sawyer, 2004).

The South African research base on responsive teaching is limited and stands at a distance from responsive teaching described in the international literature (Abdulhamid & Venkat, 2018). The nature of responsive teaching in South African primary mathematics classrooms has been recently examined qualitatively by researchers and it has been noted that evaluation of learner offers 'in the moment' are rare in the collective classroom space (Abdulhamid, 2016; Abdulhamid, 2017; Abdulhamid & Venkat, 2018; Hoadley, 2012). A consequence of not evaluating learner offers is that 'learners may well remain unaware of the extent to which their offers and narratives are "endorsable" from a mathematical perspective' (Abdulhamid, 2017, p. 200). In stark contrast to the South African research base, Abdulhamid (2017) notes that the international literature does not focus on whether learner contributions are evaluated but rather the focus is on what opportunities for learning arise as a result of the teacher's evaluation of the learners' contribution. In secondary mathematics classrooms, Brodie (2007) has illuminated that there is evidence of evaluation of learner offers which differs from the situation that currently exists at primary school level. However, the nature of teacher engagement with learners' offers is often superficial (Brodie, 2007). From our literature search, we found limited research conducted in South Africa investigating responsive teaching. Little is understood about how teachers respond to learners when faced with unexpected incidents that did not form part of their planning associated with contingency. Moreover, little is known about teachers' thoughts and decision-making when responding to unexpected events in the South African terrain.

1. By focusing on algebra, this is not to suggest that algebra has more contingent events than other topics in the mathematics curriculum. Rather, the motivation for pursuing contingency with this topic stems from our own experiences in teaching this topic and the unexpected events we encountered.

Contingency in mathematics education

In the 1970s, Alan Bishop initiated research into contingency where he was interested in how teachers deployed strategies to buy time when faced with unexpected events that caught them off guard (Borko, Roberts, & Shavelson, 2008). In recent years, there has been a surge in contingency research internationally related to how teachers respond 'in the moment' to unexpected events that did not form part of their planning (Coskun, Bostan, & Rowland, 2021; Foster, 2015; Mason & Davis, 2013; Mason & Spence, 1999; Rowland, Turner, Thwaites, & Huckstep, 2009; Rowland et al., 2015; Rowland & Zazkis, 2013). Rowland et al. (2015, p. 76) note that it is impossible for a teacher to predict all the unexpected events that can occur within a lesson but 'the teacher who anticipates such obstacles to learning is better placed to plan a less "bumpy", more joined-up learning experience for his or her students'. As previously stated, Rowland et al. (2015) have outlined that there are three triggers of contingency, namely (1) arising from learners' offers, (2) arising from teacher's insight and (3) arising from the (un)availability of a pedagogical tool.

The first trigger of contingency takes into account how the teacher responds to learners' contributions that are unanticipated and this is further subdivided into three classifications which include (1) the learner's response to a question initiated by the teacher, (2) a learner's spontaneous response to an activity or discussion and (3) the unexpected incorrect offer stated by a learner (Rowland et al., 2015). When faced with an unexpected learner contribution, Rowland et al. (2015) have proposed that teachers can decide to (1) ignore the learner's offer, (2) acknowledge but continue with the lesson or (3) acknowledge, respond and incorporate the learner's offer into the lesson. For instance, Rowland et al. recount an incident where a teacher asked learners to find the area of a rectangle with dimensions of 22 cm and 28 cm and one learner unexpectedly stated that the area would be 25 cm multiplied by 25 cm. The teacher decided to ignore the incorrect offer and continued with the lesson. Failing to acknowledge the unexpected incorrect offer denied the teacher a chance to interrogate the learner's thinking 'in the moment' and a teachable moment was missed (Boaler & Humphreys, 2005; Nesher, 1987). Also, within this first trigger of contingency, the teacher has a choice of acknowledging a learner's spontaneous response to a demonstration (Rowland et al., 2015). For example, Borko et al. (1992) present an illuminating example of a pre-service teacher attempting to construct a response 'in the moment' to a learner's spontaneous question of why one needs to 'invert-and-multiply' the second fraction when dividing a fraction such as $\frac{1}{4} \div \frac{1}{2}$. The pre-service teacher attempted a diagrammatic representation and abandoned her attempt after she realised that her representation was incorrect (Borko et al., 1992).

The second trigger of contingency emanates from teacher insight when the teacher becomes aware that something is amiss by reflecting in action during the progression of the lesson (Schön, 1987). Rowland et al. (2015) recognise that this trigger is rare among pre-service and newly qualified teachers and predict that it is more common for expert teachers. Corcoran (2008) describes a pre-service teacher in Ireland making use of butter beans to illustrate division to a class of primary school learners. The teacher designed division questions requiring learners to use a division problem structure associated with grouping. However, by reflecting 'in the moment' she senses something is amiss and realises she unintentionally guided learners to the incorrect division structure of partition. As a result of teacher insight, the teacher re-directed learners to the appropriate division structure needed to answer the division problems.

The third trigger of contingency accounts for the availability or sudden unavailability of a pedagogical tool such as technology which disrupts the trajectory of the lesson. For instance, when technology becomes unavailable, this will place demands on the teacher to deviate from their lesson plan. Rowland et al. (2015) recount an incident in a pre-service teacher's lesson where a computer package, Autograph, useful for sketching functions, refused to open. The teacher needed to act 'in the moment' and decide how to respond to this sudden unavailability of a resource. Ultimately, the teacher resorted to using the whiteboard to sketch the functions. However, it is not always the absence of a pedagogical tool that can perturb the intended trajectory. There are times that a resource can be incorporated into a lesson that did not initially form part of the teacher's lesson image. For example, a teacher in a study by Rowland et al. found a 1 to 100 square lying on a table which did not form part of her lesson plan and decided to incorporate it into the lesson based on subtraction. However, she did not realise that unexpectedly incorporating this pedagogical tool encouraged learners to respond spatially instead of symbolically which unwittingly undermined her envisaged lesson image of teaching subtraction symbolically.

In this article, we focus on the first trigger arising from learners' unexpected offers during classroom interactions. Firstly, we focus on this trigger since it was the only trigger evident across the novice and expert teachers in our study. Secondly, in the current Curriculum and Assessment Policy Statements (CAPS) in South Africa there is a press for strong pacing and curriculum coverage (Bertram, Mthiyane, & Naidoo, 2021) which has led to a situation of less attention being paid to learners' contributions during the course of teaching. Hence, more attention on this first trigger is needed to capture the subtle nuances for understanding how teachers deal with unexpected learner offers in their teaching.

What is meant by a 'novice' and 'expert' teacher?

There are differing conceptions in the literature as to what constitutes a 'novice' and an 'expert' teacher by mathematics education researchers. For instance, novice teachers are typically equated with pre-service teachers (Borko & Livingston, 1989; Borko, Livingston, & Shavelson, 1990;

Borko et al., 1992; Stockero & Van Zoest, 2013). For other researchers such as Berliner (1988), the term encompasses pre-service teachers and teachers in their first year of teaching. Moreover, Berliner goes further to categorise teachers with two to three years' experience as 'advanced beginner'. We adopt Huberman's (1993) interpretation of a novice teacher to be an in-service teacher in their first year to third year of teaching as this interpretation is broader than that of Berliner (1988) and excludes pre-service teachers.

In terms of defining an expert teacher, there is a lack of a universal consensus for what constitutes an expert teacher (Yang & Leung, 2013). Berliner (2001) notes that interpretations of expert teachers are strongly linked with the culture and contextual background in which the study is being conducted. Researchers such as Palmer, Stough, Burdenski and Gonzales (2005) note that little research has been conducted to identify a common set of criteria among expert teachers in different subject areas and these researchers identified four common criteria across different studies used to identify expert teachers. The first criterion used to select expert teachers is the number of years the teacher taught the subject which was usually at least 10 years. The second common criterion was the teacher being recommended by the school principal or colleagues as an expert teacher. The third criterion was that teachers needed to be in possession of an appropriate degree and teacher qualification. Lastly, performance-based criteria that were 'normative' and 'criterion-based' were used to select expert teachers. For our study, we found all four criteria delineated by Palmer et al. (2005) useful when conceptualising an expert teacher. If we considered only one of the criteria, this would be insufficient to describe an expert teacher. For example, if one considered a teacher in terms of years of experience yet the teacher did not have an appropriate degree this would not capture the essence of an expert teacher. We consider a teacher to be an expert if they had at least 10 years' teaching experience, were in possession of a relevant academic qualification such as having at least an honours degree and were selected to act as a mentor teacher to pre-service teachers or novices.

Several studies originating in the 1980s conducted by researchers interested in cognition and improvisation have examined novice and expert teachers on the basis of their planning, instruction and post-lesson reflection on classroom incidents (Borko & Livingston, 1989; Borko et al., 1990; Leinhardt & Greeno, 1986). In an important and highly cited study conducted by Borko and Livingston (1989), they noted that the nature of novice teachers' planning was rehearsed, scripted word for word, done at unit level and usually prepared the day before the lesson, whereas expert teachers engaged in flexible, unrehearsed, long-term planning which was done at chapter rather than unit level and expert teachers had mental plans for lessons with written planning kept to a minimum (Borko & Livingston, 1989). Borko et al. (1990, p. 43) state that the efficient planning skills of expert teachers is attributed to rich, interconnected mental schema which permit them 'to determine what information is relevant to their planning tasks and to plan more efficiently'. Moreover, expert teachers anticipated unpredictable incidents in their planning and possible learner misconceptions whereas novice teachers did not anticipate such contingencies in their planning (Borko & Livingston, 1989).

In terms of instruction, expert teachers delivered coherent lessons whereas novice teachers tended to deliver mostly disjointed lessons (Borko & Livingston, 1989). Moreover, novices had difficulties constructing explanations and making in-flight decisions when faced with unexpected learner questions as a result of their inexperience in skilled improvisation (Borko & Livingston, 1989). However, expert teachers were successful at answering unexpected questions, providing demonstrations and examples in response to unexpected learner offers as a result of their skilled improvisation (Borko & Livingston, 1989). In terms of the post-lesson reflections, novice teachers focused more on their classroom management and effectiveness as a teacher rather than key classroom incidents (Borko & Livingston, 1989). However, post-lesson reflection of expert teachers focused primarily on their learners' understanding of concepts (Borko & Livingston, 1989).

During the 1990s, researchers in Israel analysed the lessons of novice and expert mathematics teachers building upon the work of Borko and Livingston (Even, Tirosh, & Robinson, 1993; Tirosh, Even, & Robinson, 1998). For instance, Even et al. (1993) found that novice teachers delivered lessons that lacked connectedness in linking topics and ideas from earlier lessons whereas experts delivered coherent lessons that linked with ideas and topics discussed in previous lessons. In recent years, researchers have started to examine novice and expert teachers in terms of their ability to notice classroom incidents. Huang and Li (2012) examined how 10 novice and 10 expert mathematics teachers in China noticed classroom events when watching videotaped lessons. It was evident that novice teachers found noticing to be more challenging in relation to expert teachers and novices' noticing was less developed than experts (Huang & Li, 2012). Huang and Li (2012) found that:

> expert teachers paid significant and greater attention to developing mathematical thinking and ability, developing knowledge coherently, and developing high-order thinking; they also paid significant and greater attention to teachers' enthusiasm and passion, and students' participation. (p. 428)

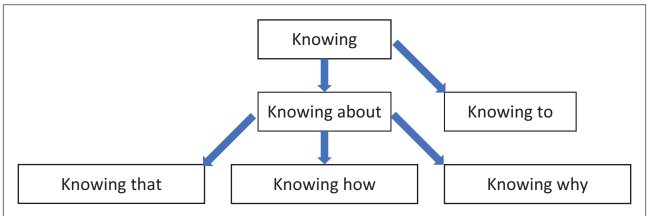
Teaching and learning of algebra

Algebra is an important branch in mathematics which serves as a precursor to other important topics such as trigonometry and analytical geometry (Watson, Jones & Pratt, 2013). However, many learners experience difficulties transitioning from arithmetic to algebra and this has been widely documented in many studies since the 1970s (Collis, 1978; Herscovics & Linchevski, 1994; MacGregor & Stacey, 1997). Common reported misconceptions include learners attempting to conjoin unlike terms as a result of the lack of finality that some expressions pose such as $b + 3$ (MacGregor & Stacey, 1997). Difficulties can also arise when teachers encourage a 'fruit salad approach' to algebra encouraging learners to let letters represent initials or labels for variables such as b for banana rather than the number of bananas (Boaler & Humphreys, 2005; Hallagan, 2006). We note that researchers in South Africa and internationally have predominantly focused on learner errors since errors and misconceptions are believed to provide a lens for understanding learners' thinking (Gardee & Brodie, 2015; Makonye & Luneta, 2014; Pournara et al., 2016; Watson et al. 2013). However, we agree with Tsui (2003) that more attention should be directed to understanding how the knowledge bases of novice and expert teachers develop and differ rather than focus primarily on learner errors.

Theoretical framework

Mason and Spence (1999) have theorised that there are two distinct forms of 'knowing', referred to as 'knowing about' and 'knowing to'. Planned aspects of teaching are related to 'knowing about' whereas 'knowing to' relates to the contingency dimension of the Knowledge Quartet. 'Knowing to' refers to active knowledge that is needed when improvising and responding to unexpected events (Mason & Spence, 1999; Rowland et al., 2015; Sawyer, 2004). Mason and Spence (1999) have categorised 'knowing about' into three divisions referred to as 'knowing that', 'knowing why' and 'knowing how' as illustrated in Figure 1. 'Knowing that' links with the foundation dimension of the Knowledge Quartet and this refers to subject matter knowledge that the teacher possesses. Mason and Spence (1999, p. 145) posit that 'knowing why' suggests being able to provide a rationale for actions performed and 'having stories I tell myself to account for something'. 'Knowing how' refers to a teacher being able to perform a procedure in front of the class such as being able to solve an equation by means of completing the square. It is pertinent to note that 'knowing about' something does not ensure that a teacher would be able to respond to unexpected events since 'knowing about' 'gives little indication of whether that knowledge can be used or called upon when required' (Mason & Spence, 1999, p. 138).

Our interest is associated with 'knowing to' since this is knowledge in action which requires improvisation. Moreover, 'knowing to' provided a suitable lens to examine interrelated concepts dealing with the phenomenon of contingency associated with teachers' responsiveness to unexpected learner offers (Figure 2).



Source: Adapted from Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38*, 135–161. https://doi.org/10.1023/A:1003622804002

FIGURE 1: Beyond 'knowing about'.

Research method and design

A qualitative case study approach was used to allow for a detailed exploration of the novice and expert teachers' responses to unexpected learner offers within their Grade 9 mathematics classrooms (Merriam, 1998). In February 2020, we observed and videorecorded three consecutive lessons of two teachers purposively selected teaching Grade 9 algebra from the same school. Both teachers were teaching simplification of algebraic expressions and distributive law when data collection took place. Prior to videorecording, both teachers were asked to prepare lesson plans for the observed lessons.

Context of the study and participants

The study is conducted in a well-resourced, private school for girls located in Johannesburg East which caters for 600 learners. Tara[2] is a novice teacher in possession of a Bachelor of Science degree and was currently in her first year of teaching when the study was conducted. Tara completed a teacher internship the previous year at the same school while registered part-time for a Postgraduate Certificate in Education (PGCE) and finished the PGCE at the end of 2020. Cassandra is an expert teacher with 20 years' teaching experience, was recommended by the Head of Department as an expert teacher and is in possession of a Master of Education. Cassandra has taught Grade 9 mathematics every year since she started teaching. The criteria used to select Cassandra met all four of the criteria outlined by Palmer et al. (2005) in the literature review. Cassandra has also served as a mentor to novice teachers over the years and is currently serving as a mentor to Tara.

Overview of data analysis process

Once data were collected, we transcribed verbatim teacher talk and teacher-learner interactions for each lesson. Our units of analysis for episodes were teachers' responses to unexpected learner offers and we coded the responses as 'appropriate' or 'inappropriate' as per Weston's (2013) coding protocol. A response was coded as 'appropriate' when

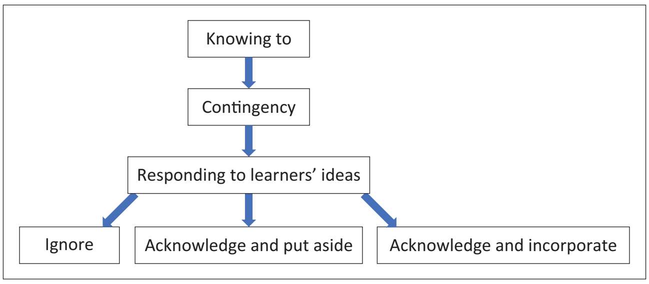


FIGURE 2: Possible teacher response pattern to triggers of contingency

2. All the teachers' names are pseudonyms intended to preserve the anonymity of all participants. No learner names are mentioned.

teachers acknowledged unexpected learner offers and these offers provided potential learning opportunities for the class. Conversely, 'inappropriate' responses included teachers ignoring or dismissing learner offers that had potential for learning and this hindered learners' access to mathematics 'in the moment'. If a response were coded as 'appropriate', we coded the quality of the response as 'minimum', 'middle' or 'maximum' as per the Weston coding protocol (Figure 3).

If a teacher chose to acknowledge a learner offer that had potential for learning but provided an incorrect response, this was coded as 'minimum'. However, if a response was correct, provided potential for learning but there was a lack of probing the learner's offer 'in the moment' we coded this as 'middle'. A response coded as 'maximum' was evident when a teacher gave a detailed explanation based on the unexpected learner offer and pressed the learner demonstrating that the full potential of the incident was exploited. Researchers note that 'press' is a fundamental aspect of the questioning process and concerns the teacher's decision to follow up on what a learner has said to understand their thinking 'in the moment' (Boaler & Humphreys, 2005; Kazemi & Stipek, 2001).

Once episodes were analysed, the first author conducted VSR interviews with each participant to help participants reflect on 'their thoughts at the time' (Mackey, 2012, p. 27) and give us insight into their decision-making in response to the unexpected learner offers they encountered. Contingent incidents were watched together with participants to help them recollect these events. Firstly, general questions were asked such as 'Did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?'. Subsequently, more specific questions related to contingent incidents were asked such as 'Can you elaborate on how you responded?' or 'If you were to repeat this lesson, would you respond to the learners' contributions in the same way or differently?' The sample of semi-structured questions during the VSR interview is included in Appendix 1.

Ethical considerations

Permission from the university (2019ECE050M) was received prior to data collection. Informed consent was obtained from teachers, learners and their guardians. Names of all participants have been kept anonymous to preserve the confidentiality of participants.

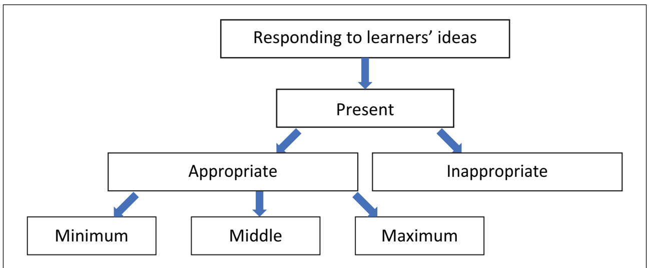


FIGURE 3: Schematic representation of coding protocol used to analyse teachers' responses to unexpected learner offers

Findings and analysis

Four contingent incidents emerged within Tara's lessons and three contingent incidents were noted in Cassandra's lessons. All contingent incidents were related to the first trigger of contingency associated with teachers' responses to unexpected learner offers and we present an overview of the findings in Table 1. As previously mentioned, if a response was present, we coded it as 'appropriate' or 'inappropriate' and we coded the response patterns that were 'appropriate' as 'minimum', 'middle' or 'maximum'. Tara chose to either ignore or acknowledge a learner offer and none of her responses was coded as 'maximum'. Conversely, Cassandra's responses were all coded as 'appropriate' and 'maximum' since she continuously interrogated learner offers and her responses provided 'maximum' potential for learning. We provide below a presentation of the dialogue between the teachers and learners and provide an analysis of each teacher's response to the contingent incident.

Tara's response patterns to unexpected learner offers

Contingent incident 1: Ignore or dismiss

In Tara's first observed lesson, a learner expresses confusion as to why $\frac{1}{x}$ can be rewritten as $1x^{-1}$ when Tara states that an expression with a negative exponent cannot be classified as a polynomial. The learner asks Tara to explain the 'exponentive one' and Tara chooses to dismiss the unexpected learner question.

Failing to acknowledge the learner's important question hindered the learners' access to mathematics 'in the moment' and we coded her response as 'inappropriate' as this question needed to be addressed. Similarly to novices in Borko and Livingston's (1989) study, Tara postponed answering the question as she was caught off guard by the question. Tara's response is unsurprising as Stockero and Van Zoest (2013) note that 18% of the response patterns of novices in their study tended to dismiss or ignore an unanticipated learner offer. Moreover, in the VSR interview, Tara acknowledged that she did not anticipate the question in her lesson plan as she thought that their previous Grade 8 teacher had taught this concept.

TABLE 1: An overview of the response patterns of Tara and Cassandra to unexpected learner offers.

Teacher	Response to contingent incident	Coding of response
Tara	Contingent incident 1: Ignore or dismiss	Inappropriate
	Contingent incident 2: Acknowledge but continue	Minimum
	Contingent incident 3: Acknowledge but continue	Middle
	Contingent incident 4: Acknowledge but continue	Middle
Cassandra	Contingent incident 1: Acknowledge and incorporate	Maximum
	Contingent incident 2: Acknowledge and incorporate	Maximum
	Contingent incident 3: Acknowledge and incorporate	Maximum

BOX 1: Tara's response to contingent incident 1.

Learner:	Ma'am can you just explain the 'exponentive' one?
Tara:	'Exponentive' one?
Learner:	Like why is it negative one?
Tara:	Let's do that tomorrow, okay? We'll spend time going through all of this stuff, okay.

BOX 2: Tara's response to contingent incident 2.

Tara:	Have fun with your calculator. Try doing all your negative exponents and try see you will still get fractions regardless.
Learner:	What if it is negative two to the power negative two?
Tara:	Negative two to the power negative two means that it is one over six.
(*Tara starts to write* $\frac{1}{6}$ *and then erases the whiteboard*)	
	Can I come back to your question? Can I come back to your question? Otherwise, you're going to get confused there.

Contingent incident 2: Acknowledge but continue

In Tara's second lesson, she spent the beginning of the lesson explaining to learners why $\frac{1}{x}$ can be rewritten as $1x^{-1}$ and then demonstrates negative exponents using a base of two. Subsequently, she uses numerical bases of three and six and asks learners to use their calculators and see that the answer will be a fraction if the exponent is negative. A learner asks Tara what the answer would be if it were -2^{-2} and Tara chooses to acknowledge the learner's question. Tara writes that the answer to -2^{-2} is $\frac{1}{6}$ instead of $-\frac{1}{4}$ and then erases the whiteboard. Tara attempted to construct an explanation 'in the moment' to an unanticipated question and realises that her attempt is incorrect similar to the novice teachers' unskilled improvisation in Borko and Livingston's (1989) study. Tara's ability to reason 'in the moment' is momentarily frozen. She refrains from further answering the learner's question by stating that she will come back to the question and continues with the lesson.

Tara's response is reminiscent of a pre-service teacher in a study by Borko et al. (1992) where the teacher attempted a diagrammatic representation not planned in advance to illustrate conceptually why one needs to 'invert-and-multiply' but abandons the attempt when she realises her answer is incorrect. We coded Tara's response as 'minimum' since she chose to acknowledge but provided an erroneous response to a learner's question which did not support mathematical learning in the collective classroom space. Stockero and Van Zoest (2013, p. 135) note that when the teacher provides an incorrect response, 'the error is likely to affect what students take away from the lesson'. In the VSR interview she admitted that 'I don't want to now go through something else where they're going to look at me so lost and confused again' and was 'scared to want to answer their questions' as they were 'not really in line with what I actually want them to know'. Tara felt she was 'going down a rabbit hole with this question' and she thought it was best to postpone further answering the question.

Contingent incident 3: Acknowledge but continue

Later in Tara's second lesson, a learner asks her 'what if it says, so x to the negative fraction?' Tara acknowledges the 'interesting question' but decides to continue with the lesson as not answering the question was likely going to have a neutral impact for the class. We coded her response as 'middle' rather than 'minimum' since her response did not introduce any errors.

BOX 3: Tara's response to contingent incident 3.

Learner:	What if it says, so x to the negative fraction? (unclear because of noise)
Tara:	Shhh … x to the negative fraction? So, square root of x to the negative fraction?
Learner:	No, just x to the negative fraction.
Tara:	x to the negative fraction? You don't really deal with that now. Can we … can we finish off and then I'll answer … cause you had an interesting question because we do deal with it in maths. You do deal with it in maths but it is going to confuse you even further so let's do what we have to do now that this is cemented.

BOX 4: Tara's response to contingent incident 4.

Learner:	Madam, isn't there an invisible one before … wouldn't it be four?
Tara:	No. Remember when one multiplies with a number, what happens to that number? The number stays the same. Right, that's what distribution means multiplying. Okay so it's positive one times two x gives me … two x right? The number is still going to stay the same. This positive, and that's three over there. *(Tara points to +3 on the whiteboard)* We are going to have three again okay. Nothing happens there and can I simplify this expression now.

We note the question asked by the learner needed to be interrogated to understand her mathematical thinking 'in the moment' as the question was unclear and needed clarity. Tara chooses to superficially engage with the question by recognising that the learner asked a 'interesting question', gives a brief response and decides to continue with the lesson as if the question did not occur. This response pattern was noted by 26% of novices in Stockero and Van Zoest's (2013) study and the authors note that novice teachers typically did not capitalise on these 'pivotal teaching moments'. During the VSR interview, she revealed that she 'was just trying to keep things as simple as possible' and was willing to 'explore these questions but not right now' as she felt she had a weak class.

Contingent incident 4: Acknowledge but continue

During Tara's third lesson, she is recapping distributive law and starts to simplify the algebraic expression: $(x+1)+(2x+3)$. Tara starts to distribute the invisible one into the first bracket and is interrupted by a learner who asks 'Madam, isn't there an invisible one before … wouldn't it be four?'

Tara chooses to acknowledge the learner's question but does not interrogate the learner's thinking 'in the moment' to understand her mathematical thinking. Tara is listening evaluatively rather than interpretively to the learner's question (Davis, 1997) and offers a procedural explanation of distributive law. We coded Tara's response as 'middle' since the response provided does support mathematical learning to a certain extent but Tara could have probed the learner with a follow-up question to get the learner to clarify and explain her thinking (Boaler & Humphreys, 2005; Kazemi & Stipek, 2001). Boaler and Humphreys (2005, p. 37) note that probing or pressing learners' thinking is important since it allows learners 'to elaborate their thinking for their own benefit and for the class'. The lack of press or probing is not uncommon among novice

BOX 5: Cassandra's response to contingent incident 1.

Learner:	I'm so confused.
Cassandra:	You are so confused. What are you confused about? All I want to know is if you understand this line? *(Cassandra points to $2x^2 + 3x - 2 - [(-x + 2) + (-5x^2 - 8)]$)* Because that's actually the hardest part … to write the polynomials whether they're added or subtracted. So, this is the hardest part. Did you get this part?
Learner:	Noooo … I don't understand.
Cassandra:	So, we need to try understand what's going on here with the words, okay. Answer these questions for me. Now you trying to translate it. Okay, listen to the words. Now you're going to translate the words into Maths. By how much is ten greater than three?
Learner:	Seven.
Cassandra:	How did you get the seven?
Learner:	Ten minus three.
Cassandra:	Right, by how much is fifteen greater than five?
Learner:	Fifteen minus five.
Cassandra:	By how much is twenty greater than seven?
Learner:	Thirteen
Cassandra:	How did you get the thirteen?
Learner:	Twenty minus seven.
Cassandra:	By how much is twenty greater than x?

teachers as a study conducted by Moodliar (2020) noted that the novice teachers in his study failed to interrogate learner's thinking 'in the moment'. During the VSR interview, the first author probed Tara's response to the learner and she knew that the learner 'wanted to jump in already and say one plus three is four' but thought it best to procedurally explain how to obtain the answer as this specific learner skips steps and does not show all her working details.

Cassandra's response patterns to unexpected learner offers

Contingent incident 1: Acknowledge and incorporate

A learner expresses confusion with the wording of the Cassandra's question which asks: By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$? Cassandra spends time trying to understand the source of the learner's confusion and realises that the learner is struggling to translate the wording into mathematics.

Sawyer (2011, p. 1) contends that 'what makes good teachers great' is their ability to engage in skilled improvisation. Cassandra is able to engage in such skilled improvisation by drawing upon her repertoire of pedagogic strategies and constructs a series of numerical questions 'in the moment' to help the learner interpret the initial question and make sense of the wording of the question. Cassandra effectively dealt with the learner's question without derailing the lesson and her successful improvisational performance is analogous to how expert teachers responded to unexpected learner offers in Borko and Livingston's (1989) study. We coded her response as 'maximum' since her response provided maximum potential for learning. When probed as to how she responded 'in the moment', it was revealed that Cassandra was able to pre-empt the question. During the VSR interview, Cassandra revealed that when a learner expresses confusion with such questions, she has learnt to 'replace all this algebra with just some integers' to serve as a 'tool that they can use when they sort of get confused with x'.

BOX 6: Cassandra's response to contingent incident 2.

Learner:	Madam, I have a question. If it's x plus five and there is a two outside the bracket?
Cassandra:	I'm getting there. Good question. So, you tell me. Give me that example.
Learner:	x plus five in brackets and then there is a two. *(Cassandra writes $(x + 5)2$)*
Cassandra:	What do you think?
Learner:	Don't you distribute the two.
Cassandra:	Why?
Learner:	Because there's a bracket.
Cassandra:	So, that is two x plus ten. So, what can you tell me about the previous example and this one?

BOX 7: Cassandra's response to contingent incident 3.

Learner:	I'm saying for the one underneath it after you've distributed it … must I write three x plus three x squared minus fifteen x or must I just write three x squared minus fifteen x.
Cassandra:	Why do you want to add another three x?
Learner:	Because it's already there … I'm just asking if I must do that.
Cassandra:	No. Because you're multiplying. So, if you're going to simplify this … *(Cassandra points to $3x(x - 5)$ on the whiteboard)* This means that three x must be multiplied. So, its three x times x. Not three x plus three x squared. You're multiplying. Did you add in extra terms?
Learner:	No, I didn't add in extra terms. I was just confused.
Cassandra:	Okay, so these two must be multiplied … *(Cassandra circles 3x and x)* And the answer to that multiplication is three x squared.

Contingent incident 2: Acknowledge and incorporate

In Cassandra's second lesson, she is teaching distributive law and works out how to simplify $(x + 5)(+2)$. A learner then asks Cassandra 'if it's x plus five and there is a two outside the bracket?' Cassandra chose to acknowledge and incorporate the learner's example into the lesson as she sees the potential of the learner's question.

Cassandra probes the learner with follow-up questions such as 'what do you think?' and 'why?' in order for the learner to clarify and elaborate her thinking. Subsequently, Cassandra asks the learner 'what can you tell me about the previous example and this one?' During the VSR interview, it was evident that Cassandra thought that learners might think that the previous example she worked out with the class of $(x + 5)(+2)$ might produce a different result from the learner's question of $(x + 5)2$. Cassandra wanted the class to make a connection and realise that the two questions yielded the same answer. Thus, deciding to deftly incorporate the example 'in the moment' proved fruitful and maximised learning opportunities for the class. We coded her response as 'maximum'.

Contingent incident 3: Acknowledge and incorporate

In Cassandra's third lesson, a learner expresses confusion as to whether the answer to $3x(x - 5)$ is $3x + 3x^2 - 15x$ or $3x^2 - 15x$. Cassandra decides to acknowledge and try to make sense of the learner's question rather than dismissing it.

Cassandra probes the learner to understand why she wants 'to add another three x' as her reasoning is unclear to Cassandra. Cassandra realises 'in the moment' that the learner wants to carry over the $3x$ since 'it's already there' and resorts to explaining procedurally that each term gets multiplied during distributive law meaning that $3x$ will not remain. Subsequently, Cassandra probes the learner as to whether she added in an extra term and Cassandra's response to the unexpected question provided maximum support for learning. During the VSR interview when Cassandra was probed on her response to the learner, she stated that 'I was thinking that she probably forgot to multiply' and reflects that 'because the last term is $15x$' the learner did not 'realise that they also have to multiply by another x which makes it the $3x^2$'. We coded Cassandra's response as 'maximum'.

Discussion

Cassandra displayed skilful improvisation in responding to unexpected events in the teaching of algebra, exhibited openness in listening interpretively to learner offers and allowed learner offers to steer the trajectory of the lesson (Davis, 1997; Sawyer, 2004). Cassandra continuously interrogated learner offers by engaging in press if a learner offer was unclear or if she wanted learners to elaborate their thinking. This proved fruitful and provided maximum learning opportunities for the class. Conversely, Tara failed to press learners when their thinking was unclear, chose to ignore or provided an incorrect answer when faced with an unexpected learner offer. This can be possibly attributed to her lack of experience when faced with unexpected learner offers and her lack of rich, interconnected mental schema to quickly access pedagogical content knowledge 'in the moment' (Borko & Livingston, 1989).

Another interesting finding was that VSR served as a useful catalyst to help Tara and Cassandra reflect on their thoughts and decision-making in response to contingent events that emerged across their three lessons. This was evident as both teachers at certain points during the interview were able to articulate a rationale for their responses to learners' questions 'in the moment'. Furthermore, the VSR interview allowed Cassandra to note potential limitations in how she responded to certain contingent incidents and envisage alternatives to how she could have responded. Cassandra recognised the potential of the VSR interview as it allowed her to consider aspects of her teaching that she did not previously consider and stated that 'it's always good to revisit and look at your teaching. We should all be recording our lessons'. However, Tara was not able to envisage improvements that could be made to her lessons and her reflections were related to her own effectiveness such as her mannerisms and classroom management during teaching similar to that of novice teachers' post-lesson reflections in Veenman's (1984) study.

Internationally, researchers have echoed concerns that teacher preparation programmes need to equip novice teachers for the intricate and challenging job of teaching through 'intellectually ambitious instruction' (Lampert et al., 2013) or 'high leverage practices' (Ball & Forzani, 2009). Barker and Borko (2011, p. 292) note that 'for the most part, novice teachers' initial opportunities to sharpen the second domain of presence – the contingent, interactive aspects of practice – do not occur until they are actually in the classroom'. We agree that many novice teachers are unprepared to engage in the improvisational act of teaching and require opportunities in teacher preparation programmes to practise how to notice and respond to unexpected learner offers. This suggests that teacher education programmes in South Africa need to consider how to prepare pre-service teachers for the demanding work of responsive teaching as novice teachers are generally underprepared in handling unexpected learner offers. This could be a potential avenue of research for mathematics education researchers.

Conclusion

Findings from the analysis indicated that the novice teacher did not press or probe learners to understand their mathematical thinking 'in the moment' when faced with an unexpected question whereas the expert teacher continuously pressed learners in all contingent incidents with follow-up questions to understand their thinking. Findings from the VSR interview indicated that both teachers were able to articulate a rationale for their decisions when reflecting on contingent incidents. Moreover, Cassandra was able to note potential limitations in how she responded to certain contingent incidents and envisage alternatives to how she could have responded. Preparing pre-service teachers for the demanding work of responsive teaching is needed and this is an avenue of research relatively understudied in South Africa.

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Authors' contributions

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Disclaimer

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Appendix 1 starts on the next page→

Appendix 1

Tara interview questions

Introduction

I would thank Tara for allowing me to have videorecorded her lesson and making time available for this interview to discuss the lesson.

Semi-structured questions

Opening conversation

a What was your intended goal for each lesson (lesson 1, 2 and 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of each lesson?

b Did you feel that you achieved what you had intended for each lesson? Why or why not?

Discussion

Responding to student ideas (Selected episodes from the lesson)

Lesson 1:

Selected episode to discuss:

a Considering lesson 1, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b In your teaching, please can you comment on how you handled a learner's contribution where she asks why there is a x^{-1} when rewriting $\frac{5}{x} - 8x$ as $5(x^{-1}) - 8x$? What were your thoughts and decision making at that moment in time when responding to the learner's question?

c If you were to repeat lesson 1, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 2:

Selected episodes to discuss:

Episode 1:

a Considering lesson 2, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b In the lesson, a learner asked a question as to simplify -2^{-2}. What was your thinking in the moment as to how to respond to the learner and what was your reason for putting aside further answering that question?

Episode 2:

a A second contingent event arose in this lesson when a learner asked how you would simplify an expression with a base x that has a negative exponent that is a fraction. What was your thinking in the moment as to how to respond to the learner's question and what was your reason for not further exploring the question?

b If you were to repeat lesson 2, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 3:

Selected episode to discuss:

Episode 1:

a Considering lesson 3, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b The class is working on simplifying $(x + 1) + (2x + 3)$ and you were working out the solution with the class. You wrote $x + 1 + 2x$ so far and a learner asks whether the answer would be four. What were you thinking in that specific moment? Please can you tell me why you responded in this way.

c If you were to repeat lesson 3, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Conclusion

a Is there anything else that you would like to add?

b What has been your experience with the three triggers of contingency *(I will explain what I mean by the three triggers of contingency to the teacher)* beyond the lessons I observed?

Cassandra interview questions:

Introduction

I would thank Cassandra for allowing me to have videorecorded her lesson and making time available for this interview to discuss the lesson.

Semi-structured questions

Opening conversation

a What was your intended goal for each lesson (lesson 1, 2 and 3)? In other words, what were you hoping that learners would be able to do, or understand, at the end of each lesson?

b Did you feel that you achieved what you had intended for each lesson? Why or why not?

Discussion

Responding to student ideas (Selected episodes from the lesson)

Lesson 1:

Selected episode to discuss:

a Considering lesson 1, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b You started answering the following question: "By how much is the sum of $2x^2 + 3x - 2$ greater than the sum of $(-x + 2)$ and $(-5x^2 - 8)$?". You wrote $2x^2 + 3x - 2 - [(-x + 2) + (-5x^2 - 8)]$ and a learner asks why they have to minus the big bracket and expresses confusion. Can you elaborate how you responded and what were you thinking as to how to respond to the learner's question? Was it an expected question?

c If you were to repeat this lesson, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 2:

Selected episode to discuss:

a Considering lesson 2, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b Cassandra, you asked the class a question regarding how to simplify $(x + 5)(+2)$ and you finished answering the question with the class. Subsequently, a learner asks you 'if it's $x + 5$ and there is a two outside the bracket?' You stated it was a 'good question'. Was this an expected question for you? What were you thinking at that moment in time as to how to respond?

c If you were to repeat this lesson, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Lesson 3:

Selected episode to discuss:

a Considering lesson 3, did you deviate in anyway whatsoever from the lesson plan you designed for this lesson?

b A learner asks you after simplifying $3x(x - 5)$ if you just write $3x$ or $3x + 3x^2$ as your answer. What was your thinking as to how to respond to her?

c If you were to repeat this lesson, would you respond to the learners' contributions in the same way or differently? If differently, please can you elaborate.

Conclusion

a Is there anything else that you would like to add?

b What has been your experience with the three triggers of contingency *(I will explain what I mean by the three triggers of contingency to the teacher)* beyond the lessons I observed?