Pythagoras - Journal of the Association for Mathematics Education of South Africa
ISSN: (Online) 2223-7895, (Print) 1012-2346

The sequencing and pairing of examples in the midst of sameness and difference: Opening opportunities to learn

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Dates:
Received: 18 Nov. 2021
Accepted: 11 May 2022
Published: 01 Nov. 2022

How to cite this article:
Pillay, V., Adler, J., & Runesson Kempe, U. (2022). The sequencing and pairing of examples in the midst of sameness and difference: Opening opportunities to learn. *Pythagoras*, *43*(1), a667. https://doi.org/10.4102/pythagoras.v43i1.667

The teaching of mathematics cannot be thought of without considering the use of examples. The examples that teachers use during a lesson and how they mediate the example set is critical to what opportunities for learning are opened up during the lesson. In this article, we explore how a teacher mediates an example set with focus particularly on what is varied and what remains the same. The case that we draw on is taken from a larger learning study conducted in Grade 10 mathematics classes and the lesson that is used in this article was the last lesson in the learning study cycle. We use variation theory, specifically how the sequencing, pairing and juxtaposing of examples provides learners with opportunities to discern the critical aspect of the object of learning. We analyse the teacher's mediation of the example set on a micro level, as this enables us to illuminate and develop our argument, while simultaneously offering a detailed example of mathematics teaching. We argue that it is the systematic and deliberate structuring of variation within an example set in the midst of invariance coupled with the teacher's mediation of both planned and learner-generated examples that is critical for opening opportunities to learn.

Keywords: critical aspects of object of learning; learner-generated examples; sequencing, pairing and juxtaposing examples; variation theory.

Introduction

Imagine thinking about and then teaching a mathematical concept (e.g. quadratic functions) without any appropriate examples (of quadratic functions). Attempting such a task would be impossible, to say the least. The use of examples forms an integral part both of doing mathematics and in the teaching and learning of mathematics. As Goldenberg and Mason (2008) have argued, examples can be seen as:

> cultural mediating tools between learners and mathematical concepts, theorems, and techniques. They are a major means for 'making contact' with abstract ideas and a major means of mathematical communication, whether 'with oneself', or with others. Examples can also provide context, while the variation in examples can help learners distinguish essential from incidental features and, if well selected, the range over which that variation is permitted. (p. 184)

Goldenberg and Mason's paper was part of a special issue of *Educational Studies in Mathematics* dedicated to the role and use of examples (Bills & Watson, 2008, p. 86). Research related to examples in mathematics education has evolved considerably over the past decade. For example, there was a special issue of *ZDM Mathematics Education* devoted to investigations on examples in mathematical thinking and learning (Antonini, Presmeg, Mariotti, & Zaslavsky, 2011) and, more recently, a special section in the *Journal of Mathematical Behavior* has focused on the roles and uses of examples in conjecturing and proving (Zaslavsky & Knuth, 2019). Across this work, as studies attended to the use and roles of examples in mathematics teaching, research on how teachers integrate examples into their teaching remained in the background (Zodik & Zaslavsky, 2008). Consequently, less was understood and known about how teachers use examples to provide their learners with opportunities to discern critical aspects of mathematical concepts, theorems and techniques.

Research on teaching and teachers' use of examples is now gaining increasing attention (e.g. Adler & Pournara, 2020; Al-Murani, Kilhamn, Morgan, & Watson, 2019; Essien, 2021).

Of interest to us is that this focus on teaching, in varied ways, brings together research on example use with considerations of variation, informed by 'variation theory' (e.g. Marton, 2015) and the illumination of structure and generality through variance amid invariance in mathematics education (e.g. Watson & Mason, 2006a). Our article contributes to this growing research field. Through a study of a selected mathematics lesson, we build an argument for the value of sequencing and pairing examples in the midst of sameness and difference in opening up opportunities to learn.

To both locate the article and illuminate our theoretical orientation to studying and working on mathematics teaching, we begin with a brief introduction to variation theory and a review of literature on exemplification with variation in mathematics teaching. We then describe the context and wider research from which we have drawn the teaching case we present. In line with Morris and Hiebert's (2011) call for the need of instructional products that are at an appropriate grain size and sufficiently detailed to inform teaching, we focus on and analyse a particular lesson on a micro level, as this enables us to illuminate and develop our argument, while simultaneously offering a detailed example of mathematics teaching.

Exemplification with variation and its significance in mathematics pedagogy

The theoretical framework for this study is variation theory (Marton, 2015; Marton & Booth, 1997; Marton & Pang, 2006; Runesson, 2005). Variation theory states that learning is a change in ways of experiencing. The way something is experienced is a function of discernment of what is to be learned, the object of learning, and particular aspects that are critical for learning something in a certain way. However, the discernment of an aspect presupposes an experience of variation of that aspect. That which is varied against a stable background is more likely to be discerned.Various studies on how variation can be used to enhance learners' learning (e.g. Marton, 2015; Marton & Pang, 2006) indicate that how the content is handled and what aspects are made possible to discern by opening those aspects as dimensions of variation in a lesson affects what is made possible to learn.

Furthermore, variation theory asserts that we learn by seeing how things differ rather than how they are similar. Therefore, to facilitate learning, contrasts need to precede similarities (Marton, 2015). When two instances are different and compared, it is possible to notice distinguishing features among them. For instance, if the intention is to help students to discriminate between integers by finding the largest number among 3, –3, –18, these examples of numbers afford additional learning possibilities compared with the examples 6, –1, 3. In the latter it is possible to give the correct answer by just looking at the digits (1 is smaller than 3 and 6). In the former, however, where two of the digits are the same (invariant) attention is drawn to the absence or presence of the negative sign (the signs vary). Furthermore, by choosing the number –18, one generally held idea from the natural numbers (18 is bigger than 3) is challenged. Hence, from a variation theory perspective, to facilitate learning, the character of the variation presented and what then is possible to notice are important.

In the same way, Watson and Mason (2006a) have argued that it is the kind of variation in the midst of what remains the same (is invariant), embedded in a set of examples presented, that is crucial for mathematics learning. Through variation amid invariance, students can come to see patterns, and to see possible underlying structure or generality across the example set, both of which are central to knowing and doing mathematics. In this article, we examine a planned example set that included various representations of linear, quadratic, hyperbolic and exponential functions, as well as learner-generated examples. We focus on their pairing and sequencing through the enactment of the lesson and how these work to open opportunities for learning.

Following the work of Watson and Mason (2002) we take *examples* to include:

> anything used as raw material for intuiting relationships and inductive reasoning; illustrations of concepts and principles; contexts which illustrate or motivate a particular topic in mathematics; and particular solutions where several are possible. (p. 4)

In discussing the place of examples in mathematics teaching, Rowland, Thwaites and Huckstep (2003) distinguish between two different uses of examples. The first use is 'inductive' and this is described as providing (or motivating learners to provide) '*examples of something*', where the 'something' is general in nature (like the quadratic function of the form: $y = ax^2 + bx + c$). Providing examples of a quadratic function (like: $y = -2x^2 - 5x + 3$) is a *particular instance of the generality*. Therefore, 'we teach a (general) procedure by a (particular) performance of that procedure' (Rowland et al., 2003, p. 86). Looking at the particular instance of the quadratic function used here, the value of the coefficient of x^2 is significant. It is not only a key feature of the function, but, more critically, if the coefficient of x^2 was set to 0, then we move out of the realm of quadratic functions and into the class of linear functions. Knowing what aspects of the 'general procedure' could change, as indicated above, is what Marton & Booth (1997) called '*dimensions of variation*'. Being aware of what values to use and what values not to use is usefully articulated by Watson and Mason (2006a) as the '*range of permissible change*'. Thus, when selecting and using examples in mathematics teaching, awareness of the dimensions of variation and the range of permissible change become significant in terms of bringing the critical aspect of the object of learning into focus with the learners. As we will see in this illustrative case, curriculum sequencing also impacts possible dimensions of variation and possibilities for range of change.

The second use of examples as identified by Rowland et al. (2003) is what they refer to as 'exercises'. This use is not 'inductive' in nature but is illustrative and intended for practice. There are additional distinctions drawn in the literature on example use. Zodik and Zaslavsky (2008) distinguish planned from spontaneous examples, where the latter arise in the course of the lesson. Watson and Mason (2005) distinguish learner-generated examples (LGEs) from those planned or spontaneous, but by the teacher, and the pedagogical value of eliciting LGEs. Our focus in this article is on inductive example use. This includes the accumulated example set (planned, spontaneous and LGEs) during a lesson and, critically, together with the teacher's mediation of the example set. We thus turn our attention to literature pertinent to this focus.

Kullberg, Runesson and Mårtensson (2013) have shown how the same task used by different teachers in a lesson levered different opportunities for learning. Drawing on principles of variation theory, they ascribed these different affordances to the manner in which the example set emerged in their lesson in terms of sequencing, aspects that were juxtaposed and contrasted, and aspects of the content that were made explicit by the teacher. In the lesson we examine, we focus on the sequencing of the examples over the lesson, with particular attention to the pairing of examples within the sequence and their enactment and mediation by the teacher.

The significance of the character of an example set in opening opportunities for learning has been reinforced by further research. Both Kullberg, Runesson Kempe and Marton (2017) and Al-Murani et al. (2019) used principles of variation to study lessons designed and taught by teachers who themselves had been introduced to such, although in different ways. Kullberg et al. studied a lesson of a teacher before and after he had participated in the processes of a learning study (Pang & Marton, 2003). They show that while both lessons were focused on linear equations, opportunities to learn that were made available were substantively different. In the first lesson, the patterns of variation of the examples drew attention to solving an equation. In the later lesson, attention was drawn to the meaning of equality and a solution – and thus very different critical aspects of working with equations. Al-Murani et al's. study took place in the context of recent changes in primary mathematics education in the United Kingdom, where lesson design has been influenced by principles of variation. They selected three lessons that were publicly available. While each lesson had a different object of learning and example set, they used principles of variation to examine what was variant and invariant, and what the teacher drew attention to. They showed how variation designed into the example sets together with the enactment of these in the lessons shaped what was made possible to learn. While these two studies had different goals, both illustrated what Al-Murani et al. describe as the 'power' of variation as an analytic tool for examining conceptual opportunities for learning in mathematics lessons. Both also showed and then emphasised how what is made available to learn was a function of both the example set and what was brought into focus (i.e. mediated) by the teacher with the learners.

In the lesson we examine, we couple a presentation of the example set with the actions of the teacher and learners as they engaged with the examples over the lesson. The lesson takes place in the South African context, and in a classroom context quite different from those in the studies discussed above. That there are merits to teaching with variation has been argued in the South African context (Mhlolo, 2013). More recently, specific studies in South Africa focused on exemplification with variation have also been reported. The recent work of Adler and her colleagues draws substantively on the notion of variation as being significant for exemplification in mathematics teaching (Adler & Ronda, 2017; Adler & Venkat, 2014). Their work turns the spotlight onto the deliberate and judicial use of examples by teachers to provide their learners with opportunities to discern the critical aspect of an object of learning by emphasising 'variation amidst invariance' (Watson & Mason, 2006a), 'sameness and difference' (Marton, Runesson, & Tsui, 2004), and simultaneity and juxtaposition (Kullberg et al., 2017). In their teacher development work, and grounded in research in the field, Adler and Pournara (2020) have worked with teachers on describing the object of learning. It is against this object of learning that they examined example sets that they have designed or taken from textbooks or from prescribed lesson plans to determine what is made possible to learn, paying attention to the sequencing, juxtaposition and pairing of examples. Their work has not extended to how these specific uses of examples might be mediated in actual classrooms.

Essien (2021) has described the use of examples in teaching as a mathematical practice that has particular import in teaching and learning in multilingual classroom contexts, and thus in the preparation of teachers for this work. In so doing, his focus on mediation of the example set is on interactional practices as these are critical for learner participation in mathematics. He brings a new dimension to this field of research by illustrating how exemplification with variation theory on the one hand, and attention to meaning making as dialogic process on the other, combine to illuminate their mutual significance in opening opportunities for learning. His empirical base is mathematics being taught in teacher education in South Africa, where multiple teaching practices are or could be simultaneously mediated. Unlike the studies discussed above, the lesson extracts analysed were not 'theory-driven based on variation theory' (Essien, 2021, p. 482), that is, designed with principles of variation theory. These can nevertheless be brought to bear on the constructed example set, and, following his concern, the patterns of interaction in their enactment. Like the other research, he shows how possibilities for learning lie not in an example set alone, but in classroom enactment. Here too, given the focus on interactional patterns, the specificity of the selection and sequencing of the examples, and how these function amid sameness and difference across the examples are not in focus.

The two research questions we thus pursue in this article are: (1) What are the sequencing and pairing of examples over the lesson, and how do these open opportunities for learning? (2) How does the teacher together with his learners act on these examples to bring the critical aspect of the object of learning in the lesson into focus? Through exploring and then answering these questions we hope to contribute to the field reviewed above through illustrating the value of sequencing and pairing of examples in the midst of sameness and difference, together with how these are enacted in the lesson, and why this mediation matters.

Research methodology and data analysis

The lesson we study in this article is drawn from a wider study that focused on teachers' mediation of a selected object of learning through their participation in a learning study (Pillay, 2013). The learning study comprised four iterations of a planned and replanned lesson across four different Grade 10 classes in two schools by four teachers who taught Grade 10 mathematics. A qualitative case study approach was adopted to examine the lessons in detail and in depth. All of the lesson plans were collected, and each lesson video recorded and then transcribed. In this article, we focus on one lesson by a teacher, Mr Nkosi (pseudonym), and specifically on his selection and sequencing of examples and their mediation.

The context

The two schools were located in the same township within the Johannesburg city municipality, an area that remains one of the poorest urban areas in South Africa. Infrastructural support is limited, and a high level of unemployment endures. Mathematics performance in the post-apartheid South African education system remains highly differentiated, with wide gaps in achievement across race and class divisions, and, to a lesser extent, within race and class divisions (Adler & Pillay, 2016). In Mr Nkosi's school, only 20% of the learners passed mathematics in the 2014 National Grade 12 final examinations with a mark greater than 40% (DBE, 2015). It is against this context, reflective of wider conditions in many South African secondary schools, and poor performance in mathematics across grades, that we tell the story of Mr Nkosi's lesson, and illustrate how he mediated the object of learning and then what eventually emerged as the critical aspect for his class of learners.

The learning study process

The object of learning for the wider learning study was to enhance learners' ability to differentiate between the linear, quadratic, hyperbolic and exponential functions across their different representations (verbal, algebraic, sets and graphical). The teachers and the researcher were aware that the object of learning as initially articulated had multiple varying aspects (class of function as well their various representations). They were nonetheless content to continue, as the lessons were targeting Grade 10 learners who had ostensibly already been taught the section on algebraic functions as per the syllabus requirements. After the second iteration of the lesson, as it became clear that learners were struggling with multiple aspects, the teachers collectively decided to refine the object of learning and focus only on the algebraic representation of the functions. In a wider context, one would not expect Grade 10 learners to display limited familiarity and fluency with algebraic expressions and equations. However, as the lessons unfolded, these difficulties became apparent[1]. Indeed, it was this narrowing of the object of learning that ultimately enabled the emergence of a critical aspect that could support learners to correctly classify the algebraic representation of a function into its appropriate class of function. It was in Mr Nkosi's lesson, the fourth lesson in the study (evidenced in detail in Pillay, 2013), that learners were provided with opportunities to discern this critical aspect and so too the more refined object of learning. Hence our choice of *this* lesson in the article.

The data

As indicated, the data included the lesson plan, and the detailed transcription of the enacted lesson. The lesson plan built on the planned example set for the wider learning study, which included examples of linear, quadratic, hyperbolic and exponential functions (Pillay, 2013). To investigate the sequencing and mediation of the example set, the transcript of the lesson was chunked into episodes. A new episode was identified when the teacher or a learner introduced a new example or a different form of the algebraic representation of the example $(\frac{1}{2}x \text{ and } \frac{x}{2})$. The examples and their sequencing are presented in Table 1.

The analytic process

Within each sequence, the examples used were examined on a detailed level from the point of view of variation. Following Watson's and Mason (2006b) argument of the importance of a systematic sequence of variation horizontally as well as vertically within the set of examples, what they call going with and across the grain (p. 4), we analysed what was constant and what varied in the examples *within each episode* as well as *between the episodes*. Finally, our focus was directed to how the content was mediated by the teacher.

Table 1 provides the lesson overview, organised into episodes (first column) by the presentation of the example set[2] (second column) We present all the examples that were used during the lesson, in the order in which they were

1.These difficulties do not only apply to this group of learners. Each year the diagnostic reports on the Grade 12 examinations, in South Africa, point to learners' difficulty in answering questions related to functions. The most recent diagnostic report based on the 2020 Grade 12 mathematics paper shows that the average performance in question 4 which was based on the rational function $h(x) = \frac{-3}{x-1} + 2$ was 55% and in question 5 which was based on the quadratic and linear functions $f(x) = \frac{1}{2} + (x+5)^2 - 8$ and $g(x) = \frac{1}{2}x + \frac{9}{2}$ was 45%. In providing suggestions for improving, the examiners suggest that teachers should spend time discussing the basic concepts of functions (DBE, 2020, p. 190).

2.We do not elaborate nor mirror here the methodology used in the wider study, as this is not pertinent to the story we tell here. Interested readers are referred to Pillay (2013) and Pillay and Adler (2015) for a detailed account of the analytical tools used to analyse the lessons, and evidence of learner discernment in the lesson.

TABLE 1: Overview of the lesson presented in episodes.

Episode	Examples	Actions by teacher (T) and learners (Ls)	Comment
1	$y = 2x$ $y = \frac{1}{2}x$	T wrote the two equations on the board, and asked the learners to identify the sameness between the two equations. Ls responded by identifying the variables 'y' and 'x' as well as the '='. T then drew the Ls' attention to the mathematical operation between the variable x and its coefficient, and Ls identified that the power of the variable x in both examples is 1. T stated that when the power is 1, the function is linear.	*Both examples are algebraic representations of a linear function. T has varied the gradient by juxtaposing 2 with its reciprocal. As will become evident below, the selection of* $y = \frac{1}{2}x$ *is important for juxtaposition and contrast in episodes 4 and 5.*
2	$f(x) = x + 1$	T put up a pre-drawn chart with the graphical (and algebraic) representations of the function. T drew Ls' attention to the graphical form and so linearity when the power of the variable x is 1.	*Remaining within the class of linear functions a third example is introduced, with a different gradient and* y*-intercept. He also varies the representation, moving from algebraic to graphical.*
3	$y = -x + 2$ $y = 2x + 2$ $f(x) = 3x + 16$	T invited learners to generate other examples of linear functions. Ls answered orally. T wrote them out on the board and then confirmed if the examples generated were linear.	*Three further examples provided by Ls remain within the class of linear functions and bring in varied gradients and* y*-intercepts.*
4	$y = \frac{1}{2}x$ $y = \frac{x}{2}$	T wrote both equations on the board, one after the other. Ls stated each equation verbally. T showed, through substitution, that the two equations yielded the same outputs for given x-values. Ls were then asked to identify other aspects that were the same. They identified the variables, the '=' sign and the power of x in both examples. Ls then classified the equations as examples of linear functions.	$y = \frac{1}{2}x$ *in episode 1 is repeated and paired with the same linear function in different equation form. The fractional forms differ with position of the variable* x*.*
5	$h(x) = \frac{2}{x}$ $f(x) = \frac{x}{2}$	T wrote both equations on the board, one at a time, asking Ls to state the equation in words. T asked learners to identify what was similar between the two equations, and they offered $f(x)$ and $h(x)$ as well as the variable x and the '=' sign as being similar. T demonstrated that $h(x)$ could be transformed into $h(x) = 2x^{-1}$ and learners then identified $h(x)$ as a rational function, a hyperbola.	*In these two examples functional notation is used: the one linear function from episode 4 is repeated and now the class of functions is varied by juxtaposing both 2 and* x *with their multiplicative inverses.*
6	$p(x) = \frac{-2}{x}$	T showed Ls a chart with $p(x)$ pre-drawn, and asked learners to rewrite $p(x)$ so that x 'does not appear in the denominator'. One L wrote $p(x) = -2x^{-1}$ and also classified it as a hyperbola.	*In keeping within the class of rational functions,* $p(x)$ *is now introduced graphically. The teacher has now varied the constant by changing it from 2 to –2.*
7	$p(x) = x^2$ $g(x) = 2^x$	T again wrote the equation of each function on the board. T asked Ls to read each example, confirmed what they said, and then asked them to identify the difference between the two equations. He drew the Ls' attention to the exponents and Ls identified $p(x)$ as a parabola.	*The quadratic and exponential function are introduced together and juxtapose the position of the variable x and 2.*
8	$g(x) = x^2 - 2$ Graph of another parabola with coefficient of $x^2 < 0$	T displayed the graph of $g(x)$ on a pre-drawn chart. T then drew a parabola with coefficient of $x^2 < 0$ on the on the same Cartesian plane as $g(x)$, thus providing another example of a parabola.	*Attention is now on the quadratic function, and three different examples presented, two in both algebraic and graphical form.*
9	$g(x) = 2x^2 + 1$ $y = x^2$ $q(x) = 2^2 + x$	T asked learners to provide examples of equations of quadratic functions. Three were offered, and T wrote them on the chalkboard and then discussed if these are examples of quadratic functions.	*As with linear functions, learners get to generate examples, and three equations are offered, two being quadratic.*
10	$q(x) = 2^2 + x$	T drew Ls' attention to the third learner-generated example and asked if this was an example of a parabola. Ls classified $q(x)$ as a linear function.	*The learner-generated example provides an opportunity to draw attention to exponent of constant vs variable.*
11	$q(r) = r^2 + 2^2$	Answering T, the L that offered $q(x)$ offered a different example, $q(r)$. T confirmed that this was an example of quadratic function and wrote it on the chalkboard.	*The learner-generated example changes 2 to variable r, but leaves the constant in power form.*
12	$p(x) = x^2$ $g(x) = 2^x$	T pointed back to equations in episode 7 that were on the board and highlighted the difference in exponents. T asked the learners: if the variable x is the exponent, then what type of function is it? Ls classified this as an exponential function.	*Quadratic and exponential function in which the position of the variable* x *and 2 are juxtaposed – as previously introduced. Both are in algebraic form.*
13	$h(x) = 2^x$	T displayed pre-drawn graph of $h(x)$. T now drew Ls attention to the shape of the four pre-drawn graphs, each drawn on a chart and pasted on the board: $h(x) = 2^x$; $f(x) = x + 1$; $p(x) = \frac{-2}{x}$ and $g(x) = x^2-2$.	*The graphical representation of the exponential function is presented. Four different functions each in both algebraic and graphical form are now displayed on the board, and so fusion … in relation to the powers and of the variable x.*
14	$y = 3^x + 2$ $g(x) = 3^x - 1$	T invited Ls to generate further examples of exponential functions. Ls called out equations. T confirmed that these were examples of exponential functions and wrote them on the chalkboard.	*Learner-generated examples of exponential functions in which they vary the constant.*

used. This presentation enables us to illustrate Mr Nkosi's deliberate sequencing and pairing of examples. In the third column we describe the teacher and learner actions in the specific episode and in relation to the example set. In the commentary in the last column for each episode we draw attention to how Mr Nkosi used the selected examples to draw attention to sameness and difference, thus providing learners with opportunities to discern critical aspects of a selected object of learning.

Following the presentation of the example set in Table 1, the detailed analysis, and so findings of the study, are discussed in two sections: the first focused on the sequencing, pairing, juxtaposition and simultaneity of the examples, and the second on the teacher's and learners' actions and so the teacher's mediation of the example set, and of LGEs.

Sequencing, pairing, simultaneity and juxtaposition of examples

Taking an overall look at the examples in Table 1, we can see that the representation of the functions varied (algebraic and graphical) and the set of examples is restricted by the use of the number 2, but with varying positions in the functions. This is probably critical for the possibility to learn characteristics of a class of functions. However, what we find most interesting is the pattern of variation and invariance that unfolds in the example set when variation is introduced against a background that remains stable.

As noted, following Watson's and Mason (2006b) argument of the importance of analysing with and across the grain (p. 4), we examined what was constant and what varied in the examples *within each episode* as well as *between the episodes*. What was found significant and reoccurring in this lesson is, on the one hand, the juxtaposition and contrast of pairs of examples and, on the other, how the change to a new class of functions happened by keeping something invariant while varying something else.

Pairing and contrasting – going across the grain

As can be seen in Table 1, reoccurring throughout the episodes in the lesson is the pairing of examples. These chosen examples bring out differences and similarities that make it possible to notice features of classes of functions and thus to distinguish one class from the others.

In the first set of episodes (1–4) only the linear function is in focus with various examples presented on the board. So, the class of function is the same while the examples vary. Looking at what is different and thus what is compared within each pair of examples, the gradient varies in episode 1 ($y = 2x$ and $y = \frac{1}{2}x$), while in episode 4 the fractional form of the function is different ($y = \frac{1}{2}x$ and $y = \frac{x}{2}$) and in episode 3, the gradients and intercepts vary (the LGEs).

In episode 5 a new class of functions is introduced: rational functions. What was the same in the previous episodes (the power of $x = 1$) is now changed in episode 5. A pair of examples, one of a linear function ($f(x) = \frac{x}{2}$), the other of a rational function ($h(x) = \frac{2}{x}$), is used to bring out a contrast between linear and rational functions. These examples are then compared; what is similar and different? By varying the position of x and 2 in the two examples (i.e., juxtaposing 2 and x with their multiplicative inverse) and varying the algebraic form of $h(x) = \frac{2}{x}$ into $h(x) = 2\,x^{-1}$, the power of x in a linear versus a rational function is made possible to notice.

In episodes 1–4 and 5–6 only one class of function was handled at a time and the pair of examples taken were examples of the same class of functions. In the next set of episodes (7–13), however, two classes of functions (exponential and quadratic) are discussed and handled simultaneously. First (episode 7) a pair of examples, one quadratic, one exponential, is juxtaposed ($p(x) = x^2$ and $g(x) = 2^x$). Within this pair of examples, the exponent varies (exponent = 2 and exponent = x) and, subsequently, the position of the variable is also varying. After having compared these two examples, in the next moves (episodes 8–11) the class is restricted to only one class (quadratic), but with various examples of quadratic functions presented on the board. However, one of the LGEs is not a quadratic function (episodes 9–10). This variation was brought in by a learner, and the teacher uses and compares this example (linear function) to the other quadratic functions. Again, examples are juxtaposed and contrasted.

Sequencing and juxtaposing – going with the grain

Looking at how the content is presented in terms of the sequencing of the different classes of functions, it may look like 'taking one thing at the time', and that the learners should master one thing before learning another. However, a closer look at differences and sameness concerning the examples when a new class of functions is introduced (i.e. what changes vertically) tells us that is not the full picture. Instead, the analysis shows how the move from one class to another is carefully done using variation against a stable background.

This can be seen, for instance, in the move from episode 4 to episode 5 when a new class of functions (rational functions) is introduced. What was the same (the power of x) in the previous episodes is now changed. In this switch, a pair of examples is present on the board. One of the examples ($f(x) = \frac{x}{2}$, a linear function) is the same as in episode 4. The other one is a new example ($h(x) = \frac{2}{x}$, a rational function). These examples are then compared: what is similar and different? In this way, in the move from episode 4 to episode 5, when introducing a new class of functions, one of the examples of linear functions, which the learners have demonstrated to be familiar with, is picked up and used as a link to a new class in episode 5. This pair of examples serves as a contrast between the two classes of functions. By comparing the examples, the difference in relation to the power of x is made possible to discern. In this way of contrasting examples with x^1 to examples of x^{-1}, the teacher is weaving connections (Ekdahl, Venkat, Runesson, & Askew, 2018) between episodes 1–4 and episodes 5–6 and, thus, the classes of functions.

Similarly, in episode 6 the teacher uses an example from the previous episode 5 ($h(x) = \frac{2}{x}$) but changes the constant from positive 2 into negative 2 ($p(x) = \frac{-2}{x}$). In this way, another example of a rational function (hyperbola) is presented in two different forms ($p(x) = \frac{-2}{x}$ and $p(x) = -2x^{-1}$).

Another example of 'returning' and weaving in examples between episodes to bring out differences and similarities, is seen in episodes 7–12. In episode 7 the examples ($p(x) = x^2$ and $g(x) = 2^x$) are juxtaposed and contrasted (quadratic vs exponential); thus, the variation concerns two classes of functions. In episodes 8–11 only one of them is in focus (quadratic). Thus, the class of functions is the same. However, in episode 12 the pair of examples ($p(x) = x^2$ and $g(x) = 2^x$) used was discussed and contrasted in episode 7, hence it is picked up and contrasted once again. However, whereas the comparison of the exponent in episode 7 resulted in a focus on quadratic functions, the comparison in episode 12 led to the exponential function coming into the fore of attention.

This way of returning to an example previously discussed, is similar to what happened in the move from episode 4 to episode 5. Here the example ($y = \frac{x}{2}$) from episode 4 made up a pair together with a new example ($y = \frac{2}{x}$).

To conclude, there is a systematic pattern of variation and invariance in the sequence, the juxtaposition and contrasting of pairs of examples and in the moves between the different classes of functions. This we infer, from a variation theory point of view, afforded the learners opportunity to experience and discern the different powers of x and the different positions of the exponent.

The teacher and learner actions on the example set – its mediation

As we have argued, the example set on its own, no matter how systematically organised, also needs to be mediated. We thus turn out attention now to excerpts of teacher and learner actions.

Mr Nkosi's mediation of the example set

It was shown in the previous sections that the example set was made up of pairs of juxtaposed examples. Once the examples were introduced, Mr Nkosi asked learners to *compare* the given pair of equations. Here the questions 'What is same? What is different?' guided the learners' attention to critical aspects of the class of functions. The results of our analysis suggest that this comparison was made in a shift from emphasising sameness to differences and vice versa. We will use two episodes about linear functions to illustrate this.

In episode 1 the two equations $y = 2x$ and $y = \frac{1}{2}x$ were compared. It was concluded that they were different regarding how they were read and written algebraically. So, differences between the equations were in focus. Next, what they had in common was attended to. The students suggested: 'y', 'x' and '=' as common features. The teacher agreed and added 'they are all equations' but pointed out a significant difference – the coefficients: 'two times x' and 'half times x'. Mr Nkosi concluded: 'So, you'll find, as you have already indicated, that's something in common'. He circled the unknown x in each equation, and continued: 'You've got two being multiplied, and again here you've got half being multiplied by x'. However, he leaves this difference and comes back to what is similar – the exponent is 1:

Mr Nkosi	If I could ask you again, what is the exponent of x in both equations? Yes girl?
Learner	One

In the same way, in episode 4, by the questions he asked, Mr Nkosi elicited differences and sameness between the examples of equations. He initially asked the learners to read the examples $y = \frac{1}{2}x$ and $y = \frac{x}{2}$. It was then concluded that they were different regarding how they were read and written algebraically:

Mr Nkosi	No, they don't sound the same. y is equal to half x, and y is equal to x over two. We're saying they don't sound the same. And hence they are not the same.

So, these differences between the equations were in focus. Next, what they have in common was attended to by lifting a feature that was not instantly visible in the equations – the value of x. He substituted the x-values and demonstrated that although the algebraic expressions were different, they yield the same value and were the same in that respect. He asked: 'Are they the same?' and concluded:

Mr Nkosi	They are written differently but it is …?
Learners	The same.
Mr Nkosi	The same.

In both episodes he drew the learners' attention to sameness which goes beyond the superficial level and irrelevant commonalities (they are written differently or sound different) and eventually ended up with that which was the critical aspect – the exponent.

When in episode 1 Mr Nkosi asked about differences between the two examples of linear functions – how they were read and written – a contrast between visible features of the equations was made. Once these differences were identified, the teacher shifted and asked about sameness and concluded that the exponent was the same (1). The learners' answers (x, y, =) indicated that these were the features they actually discerned. They did not notice that they both were examples of linear functions. This can be interpreted as attending to superficial, not essential, features of the equations. However, these are obviously what the learners paid attention to. When being asked about commonalities, no one (audibly) answered that the class of function was in common. Neither did they notice the difference between coefficients (multiplying by two vs multiplying by one half).

These examples demonstrate how questions of sameness and differences supported the character of the examples in the pairs. As was described previously, the set of examples was chosen so that something changed against a stable background between and within the examples. This was mediated by the teacher's questions and actions which afforded opportunities for the discernment of the critical aspect. Following principles from variation theory, stating that learning is a matter of differentiation, we would suggest that by asking about sameness and differences and successively comparing features of the equations that are not critical and, thus, can be neglected, focus on the critical aspect of linear functions (the exponent of x) emerged and came to the fore.

Mr Nkosi's mediation of learner-generated examples, and so of learner actions with examples

Watson and Mason's (2005) idea of LGEs, which is essentially a process where the teacher invites learners to generate examples according to specified features, is not a typical practice in South African mathematics classrooms. They argue that learners who consistently employ example generation as an integral part of their learning strategy undergo more shifts of concept image, provide better explanations, develop broader example spaces and have a more complete understanding of the taught concept.

In episodes 3, 9 and 14 we observe that Mr Nkosi asked his learners to generate their own examples of linear equations (episode 3), quadratic equations (episode 9) and exponential equations (episode 14). By engaging his learners in the process of generating their own examples he provided them with some opportunity to assess their own understanding and it also provided him with some insights as to whether the critical aspect had come into focus for his learners. Of particular interest here is the LGE $q(x) = 2^2 + x$ (episode 9) as an example of a quadratic function. Once it was confirmed that $q(x)$ is an example of a linear function, the learner who generated $q(x)$ had a follow-up question. The extract below is from the lesson transcript that deals with Mr Nkosi's engagement with this follow-up question:

Learner	Can I ask a question? Is it … must it always have an x to an exponent two?
Mr Nkosi	Right, there is the question. Must it (*teacher emphasises 'it'*) always have an exponent two?
Learner	Yes.
Mr Nkosi	You got the answer?
Learner	Must x always be having an exponent two?
Mr Nkosi	Must this always be an x having an exponent two? Right, that's her question. Must it always be an x here? Right. (*erases* $2^2 + x$) We are saying x squared plus two to the exponent two (*writes* $q(x) = x^2 + 2^2$). Must there always be an x here (*points to* x^2)?
Learners	Yes.
Mr Nkosi	You are saying?
Learners	Yes.

And continuing with his attention to variation amid invariance and the critical aspect of the exponent of the variation, Mr Nkosi followed up further:

Mr Nkosi	Yes, how about if we have it as q of x (*writes* $q(x)$) … sorry, (*erases* $q(x)$) q of r equal to r squared plus two squared (*writes* $q(r) = r^2 + 2^2$). Is this not a parabola as well?
Learners	It's a parabola.
Mr Nkosi	It's a parabola. Because the variable here, the highest part of the variable is still two. Ok? So it doesn't restrict us to an x. It depends on which variable we have chosen.

The example generated, $q(x) = 2^2 + x$, illustrates that what has come into focus for the learner is that if one of the terms in an equation is squared, then the equation represents a quadratic function. Out of focus for this learner is the critical aspect that the independent variable has to be squared in order for the equation to represent a quadratic function[3]. By allowing this learner to create her own example of a quadratic function Mr Nkosi provided opportunities for the learner to compare her understanding of the concept in focus with that which is accepted as valid. He reinforced the critical aspect being the independent variable by varying the letter used to symbolise the variable. His mediation thus provided this learner with the opportunity to develop mathematical meaning by experiencing structure and extending the range of variation which contributes to experiencing generality (Watson & Mason, 2002).

Mr Nkosi's use of LGEs across the lesson also provided him with a yardstick by which to measure, to some extent, the stability of his learners' discernment of the critical aspect before varying another aspect of the object of learning and thereby moving into another class of algebraic function as is seen in the transition from episodes 3–4 to episode 5 (from linear function to rational function) and again from episodes 9–11 into episode 12 (from quadratic function to exponential function).

To conclude our presentation of Mr Nkosi's mediation of the example set, he consistently drew learners' attention to what was the same and what was different within pairs of function equations, with particular attention on the exponent of the variable; by inviting LGEs he created opportunities for learners to not only participate and produce particular function equations, but also for him to be able to mediate where and when necessary.

Concluding discussion

We revisit the research questions that guided our study viz. (1) What are the sequencing and pairing of examples over the lesson, and how do these open opportunities for learning?; (2) How does the teacher together with his learners act on these examples to bring the critical aspect of the object of learning in the lesson into focus? To answer these questions, we analysed Mr Nkosi's lesson on a micro level focusing on both the selection of examples used during the lesson and how he mediated the example set, which includes his mediation of LGEs. Using principles of variation theory, we examined the example set used and illustrated the value of sequencing, pairing and the juxtaposing of examples in the midst of sameness and difference to make the identified critical aspect discernible. In so doing, we further illuminated variation theory as being a practical analytic tool by which one can gaze into the teachers' use of examples and contribute to theorising teachers' work. Looking at the selection of examples through the lens of variation theory alone was

3.The Grade 10 syllabus restricts work on the quadratic function to having the axis of symmetry at $x = 0$: vertical transformation and reflection along the x-axis. Thus, in this lesson only examples of the form $y = ax^2 + c$ are explored.

insufficient for us to determine how opportunities for the learners' discernment of the object of learning were made possible. To gain some understanding here, we also looked at how Mr Nkosi mediated the example set that was created during the lesson and concluded that it is the example space together with its mediation that is of significance in providing opportunities for the learners' discernment of the object of learning. We have shown both that and how his questions, revolving around sameness and differences, focused and helped to draw attention to the pattern of variation within and between the example sets. We thus conclude that it is the example space together with its mediation that is of significance in providing opportunities for the learners' discernment of the object of learning.

Furthermore, the study gives an empirical illustration and supports Watson and Mason's (2005) suggestion of the pedagogical value of eliciting LGEs. We have demonstrated how the teacher picked up and used a learner's example (even if incorrect) and how these examples added to and expanded the pattern of variation. In this way our study contributes to extending previous research on variation theory designed and analysed lessons (e.g. Al-Murani et al., 2019; Kullberg et al., 2013). This article also contributes to the literature of how teachers integrate examples into their teaching by illustrating how a teacher deliberately *sequenced and paired examples* that featured *variation amid invariance*, while *explicitly* bringing sameness and difference to the learners' attention through *simultaneity and juxtaposition*, and *contrast*. This enabled learners to discern a critical aspect of an object of learning, specifically distinguishing classes of functions. Furthermore, it is clear from the analysis of the lesson that it is a systematic pattern of sameness and difference between the examples that characterise their choice and mediation. This was a deliberate attempt by Mr Nkosi to bring the degree of each of the equations as represented by the various examples into focus for the learners. Hence, we have demonstrated how a teacher can make use of principles of variation and invariance for planning the sequence of examples in the lesson and mediate the features of the examples in a way that makes them possible to discern.

Our analysis of Mr Nkosi's lesson also contributes to the growing research in the field by describing in detail (i.e. at a micro level) how a teacher used examples to provide learners with opportunities for the discernment of an object of learning, and more specifically to research in South Africa by studying a teacher's enactment heeding the call by Morris and Hiebert (2011) who argue for the need of instructional products that are minute and significantly detailed to inform teaching. We also agree with Essien (2021) that using examples is a mathematical practice and that doing mathematics entails exemplifying. Thus, when extending this to teacher education, the use of principles of variation theory plays a central role in the mathematics teaching practice of using examples. This also resonates with Adler and Pournara's (2020) framework which illustrates that teachers need to do more than just use examples; they need to use examples deliberately.

We are not arguing that teaching with variation or comparing and contrasting by using examples is novel or unique. What we are arguing for is that it is not just any variation or any contrasting that matters, but a systematic and deliberate variation in the midst of invariance that is critical, or more specifically how something is changed against a stable background. In our study we have shown *what* variation and *what* sameness, the characteristics of the variation, and *what* is juxtaposed and compared are significant from the point of view of possibilities to learn. In this case the variation concerned the critical aspect.

Zooming into Mr Nkosi's lesson we note that the relationship between the equation and so algebraic representation and its graphical representation of different classes of functions was brought into focus as these graphs were drawn. However, this was not simultaneously in focus, indicating that Mr Nkosi's goal was to focus first on distinguishing the algebraic forms. Focusing on the critical aspect that emerged during this learning study we note, mathematically speaking, that $y = 2x$ is not linear because it is a polynomial function of the first degree. It is linear because it has a constant rate of change; $y = x^2$ is quadratic because the rate of change of the rate of change is a constant and not that it is a polynomial function of the second degree. In terms of relating the syntax of the algebraic representation of a function to its graph, focusing on the value of the exponent of the independent variable provides some criteria for learners to able to identify the class of function represented by a given equation. It is precisely the exponent of the independent variable that emerged as a critical aspect for this group of learners as it provided them with some 'rules' by which to recognise the class of function given its algebraic representation. One may argue that this critical aspect is a visual cue which is insufficient since it is not grounded in mathematics. This critical aspect may thus be inadequate for the learners' understanding of functions, and may lead to learners developing partially formed ideas and possibly what literature refers to as prototypical thinking (Tall & Bakar, 1992). Although what emerged as the critical aspect in this study may be inadequate for learners developing a deeper understanding, it was nevertheless crucial for them, since it provided them with some resources with which to go forward in terms of their learning of mathematics.

Of course, we are not suggesting that their experience of learning about classes of functions remains at this level of thinking. Our goal in this article was not specific to the learning and teaching of functions. It was rather to illustrate the value of sequencing and pairing of examples in the midst of sameness and difference, and the opportunities this can open for learning. It is a value, we submit, that could inform further research and practice in different mathematical domains and topics, and at different grade levels in the curriculum. Implications for mathematics teacher education follow: prospective and practising teachers can themselves experience the value of deliberate exemplification as a mathematical and a mathematics teaching practice.

Acknowledgements

Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

This article is based on a larger study conducted by V.P. and therefore V.P. collected the data. V.P., J.A. and U.R.K. contributed by writing the article, reviewing the literature and analysing the data. J.A. and U.R.K. contributed to the conceptualisation of the article and guided the discussions on the analysis of the data.

Ethical considerations

Since human participants (teachers and their learners) as well as schools were involved in this study, the study adhered to ethical research principles. The relevant ethics clearance was obtained from the university general/human research ethics committee (2011ECE005C).

Funding information

This work is based on research supported by the South African Research Chairs Initiative of the Department of Science and Technology and the National Research Foundation (Grant no. 71218).

Data availability

Data sharing is not applicable to this article, as no new data were created or analysed in this study.

Disclaimer

Any opinion, finding and conclusion or recommendation expressed in this material is that of the authors and the National Research Foundation does not accept any liability in this regard.

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